DOCUMENT RESUME

ED 259 897 SE 045 863

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TITLE Guidelines for Grades 9-12 Mathematics Curriculum. Toward Meeting Present and Future Needs.
INSTITUTION Washington Office of the State Superintendent of Public Instruction, Olympia, WA. Div. of Instructional Programs and Services.
REPORT NO IPS-624-85
PUB DATE May 85
NOTE 72p.
PUB TYPE Guides - Non-Classroom Use (055)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Algebra; Critical Thinking; *Curriculum Development; Curriculum Guides; Functions (Mathematics); *Geometry; High Schools; Mathematics Education; Measurement; *Problem Solving; *Secondary School Mathematics; *Vertical Organization
IDENTIFIERS *Washington

ABSTRACT

Three sequences of coursework are detailed in the curriculum development guidelines provided in this document. The 4-year sequence, structured around problem-solving, applications, and the acquisition of theory, is designed for the college-bound student who plans to enter a mathematics-based field of study. The 3-year sequence is designed for students whose present plans are not directed toward entrance into a mathematics-based college program. Although this program is not as mathematically rigorous as the 4-year sequence, students who complete the coursework in the sequence will be prepared (should their plans change) to enter college-level precalculus. The 2-year sequence is designed for students whose present progress in mathematics, achievement level, and aspirations preclude the potential for success if enrolled in either of the other sequences. This sequence provides for meeting the minimal state graduation requirements in mathematics. For each of the sequences, the guidelines indicate the suggested years in which topics should be introduced, developed, and mastered. Major topic areas for the three sequences are: critical thinking and problem-solving; measurement; geometry; number properties, theory, and computation; algebra; and functions. Introductory comments, statement of philosophy, and recommendations upon which the document rests are also provided. (JN)

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GUIDELINES FOR GRADES 9-12 MATHEMATICS CURRICULUM

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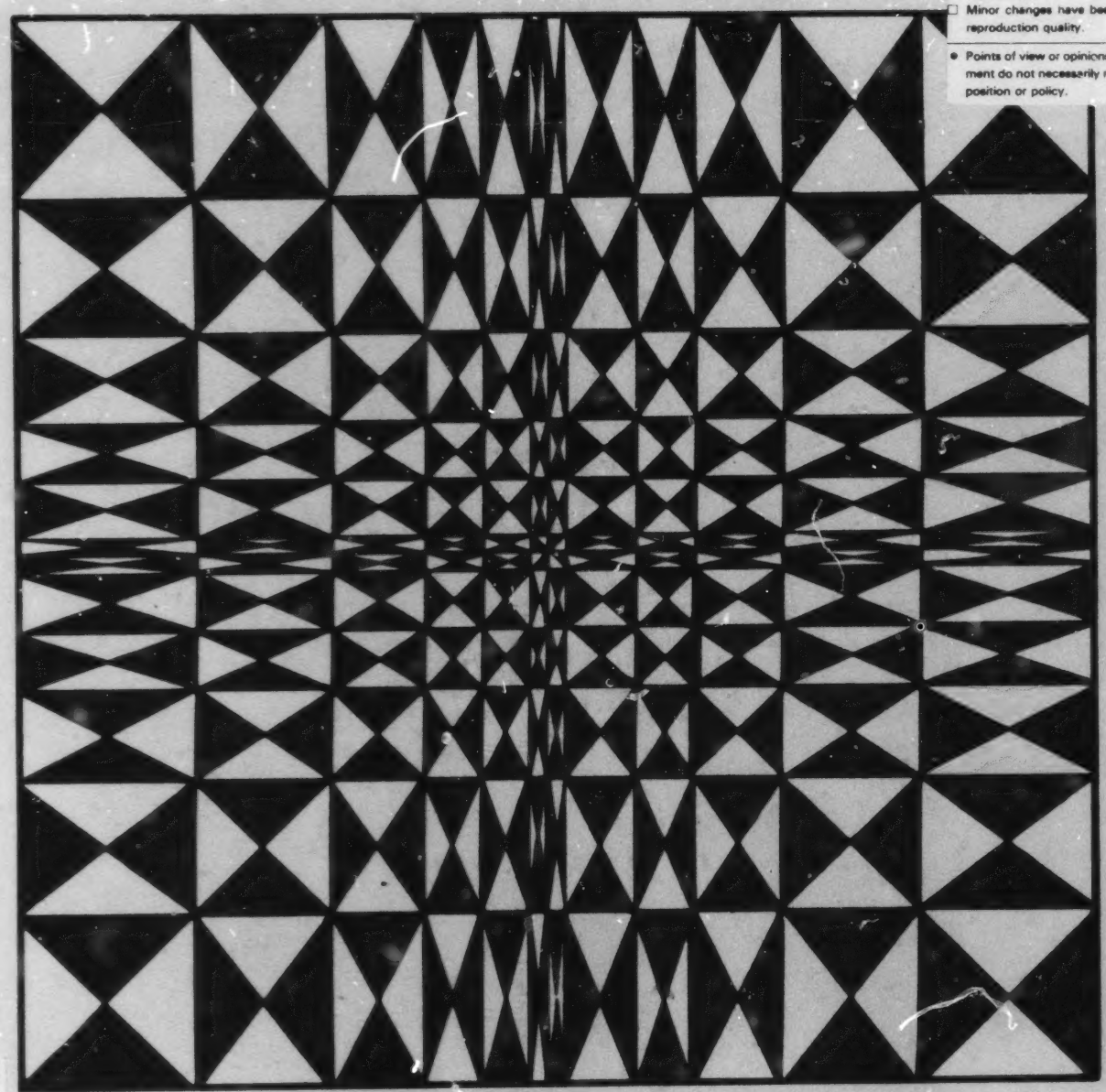

Toward Meeting Present and Future Needs

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GUIDELINES FOR 9-12 MATHEMATICS CURRICULUM

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May, 1985

SPI

DR. FRANK B. BROUILLET — Superintendent of Public Instruction

MESSAGE FROM THE SUPERINTENDENT OF PUBLIC INSTRUCTION

As society moves toward the twenty-first century there will be a need for all citizens to become prepared for an increasingly technological age. Certainly our schools must point the preparation of our K-12 students in this direction. To this end the State Board of Education adopted in May 1983, increased graduation requirements and requested my agency to prepare program suggestions and curriculum guidelines to match the graduation requirements. The State Board also stipulated that the guidelines should reflect the desire to achieve excellence across both academic and vocational areas, and to prepare students with the skills required for college and work.

It is my hope that school districts will find these mathematics guidelines, the first in the series of curriculum guidelines to be developed, helpful as they upgrade curriculum, revise and complete Student Learning Objectives, and engage in other program improvement efforts.

As we proceed then with joint efforts to upgrade mathematics curriculum, I strongly urge that you adopt a dual thrust of equity and excellence which will:

- facilitate improvement and change in mathematics curricula
- enhance the quality of teaching in these fields
- increase the enrollment of students, particularly women and minority males, in mathematics.

The implementation of a plan to develop the Mathematics Curriculum Guidelines resulted in the involvement of many educators from all levels throughout the state. Many hours have been given in order to reach concensus on these guidelines. I congratulate the educators whose names appear on the pages of this document for their excellent work in developing a set of guidelines for grades 9-12 mathematics curriculum. The ideas and concepts expressed herein will truly lead us forward in meeting present and future needs in mathematics education.

Sincerely,

Frank B. Brouillet

Frank B. Brouillet
State Superintendent
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INTRODUCTION

The High School Core Mathematics Writing Group was directed by the Office of the State Superintendent of Public Instruction to prepare a set of curriculum development guidelines for high school mathematics. This resulting document is intended to serve as a basis for the development of high school mathematics curricula by local school districts to meet current demands and serve future needs. It is anticipated that additional topics will assume importance in the mathematics needs of high school students and will be incorporated into the framework as required. Conversely, topics of declining importance will continue to be phased out.

Three sequences of coursework are detailed in this document. Each will satisfy state graduation requirements. They are differentiated by both the extent and the depth of content coverage.

I. Four-Year Sequence of integrated study is designed for the student who plans to enter a mathematics-dependent college program. (It should be noted that coursework in the social sciences and other areas not traditionally associated with mathematics are becoming increasingly mathematics-dependent.)

II. Three-Year Sequence of integrated study is designed for the student whose present plans are not directed toward entrance into a mathematics-dependent college program.

III. Two-Year Sequence of integrated study of high school mathematics for the student whose present progress in mathematics, achievement level and aspirations preclude a potential for success if enrolled in either of the other sequences. This sequence provides for meeting the minimal state graduation requirements in mathematics.

The sequence framework allows for sufficient flexibility so that students may move from one sequence to another as plans change and needs dictate. It should also be noted that an "honors track" program for exceptionally able students in mathematics has not been detailed in this document. (See item F, page 7.)

The members of the High School Mathematics Core Writing Group feel strongly that the curricular reforms with which this document is concerned require a commitment on the part of the responsible agencies, including those at the state and local levels, to provide the financial resources and leadership that are essential for their successful development and implementation. These resources must provide for an increase in the number and quality of mathematics classes available to students as well as meeting curriculum development and teacher training needs.

PHILOSOPHY

Today's and tomorrow's students will be living in a world that is technologically based and in a society whose complex problems cannot be solved with yesterday's tools. To contribute to and survive in such a world, students will need fluency in mathematics that goes well beyond the back-to-the-basics/computation emphasis of recent years. All students should have the opportunity to gain as strong a preparation in mathematics as their abilities will allow in order to give them the opportunity for maximum intellectual growth and for an optimum range of career choices.

Traditionally, the approach to the instruction of high school mathematics topics has been based on isolated skills development. This has led to a situation wherein the most basic mathematics skill of all, critical and insightful thinking, is almost entirely overlooked. The result is that students learn mathematics as a largely unrelated mass of facts with little real meaning or purpose; they are woefully deficient in their ability to think creatively and imaginatively in mathematical situations; and they have difficulty formulating and/or solving any but the most routine mathematical problems.

Both the National Council of Supervisors of Mathematics and the National Council of Teachers of Mathematics have given first priority to problem solving and the development of problem-solving skills in their recommendations for needed changes from current practice.* We are in total agreement.

Problem solving is more than just getting an answer to a word problem. It requires that students apply their mathematical skills and understandings, inquiry skills, and thinking skills to formulate and solve problems. The National Assessment of Educational Progress (NAEP) and other studies show that our students do comparatively well in mathematical situations requiring only the use of the lower-level cognitive skills; but the same studies show that students do dramatically less well in dealing with mathematical situations that require problem-solving skills.

The members of the High School Core Mathematics Writing Group believe that:

- A problem-solving approach has the potential for greater excitement and motivation than isolated skills learning. By making problem-solving activities the mainstay of instruction and learning, students' abilities to think analytically will improve, they will develop a more positive attitude toward the study of mathematics, and their command of the traditional skills will develop as a natural consequence.

*See the NCSM Position Paper on the Ten Basic Skills Areas in Mathematics, and the NCTM An Agenda for Action - Recommendations for School Mathematics of the 1980s.

- A problem-solving approach encourages students to experiment with a variety of methods in the process of reaching solutions. Recognition given to students for their selection of strategies/methods will give them additional incentive for the further development of their problem-solving skills. In this regard, it is important to keep in mind that the processes that are used are at least as important as the solutions that are obtained.

- A problem-solving approach is every bit as important with regard to meeting the needs of low-achieving students as is the case with their more able peers. These students need many of the same problem-solving skills as do students in more advanced mathematics courses in order to function adequately in the world in which they must live. The use of problem-solving activities, at suitable levels of sophistication, involving a wide variety of situations and manipulative experiences, will often motivate low achieving or "turned off" students, resulting not only in improved skills and analytical abilities, but also in a lessening of discipline problems, in a heightened interest in learning, and in improved self-perception.

- A problem-solving approach provides a natural setting for consideration of "real life" topics including those from business, industry, and other vocationally-oriented sources. In turn, this helps to respond to the often repeated question, "When are we ever going to have to use this?" This could supply the additional spark that is needed by some students to begin to take the study of mathematics seriously.

- A problem-solving approach makes possible the natural integration of topics from the courses in the traditional sequence of high school mathematics, allows for ease of inclusion of "new" topics as they surface in importance, and the deletion of "old" topics as they fade in importance. It also provides for spiral review for a flexible pace in the introduction and development of topics.

In summary, problem solving should not only be the first priority throughout the high school mathematics curriculum, but it should also be the vehicle that "carries" the instruction/learning in skill and concept development. The regressive "back-to-the-basics" movement, characterized by a minimum competencies emphasis and restricting attention almost exclusively to the computation skills, must be put to rest once and for all. We believe that all students are capable of becoming problem solvers with the range and depth of the development of the associated skills being dependent only upon their potential for learning.

RECOMMENDATIONS

A. Problem-Solving Approach

The basic recommendation on which this document rests is that problem solving must be the primary focus of mathematics instruction/learning throughout the high school mathematics program. To accomplish this goal, there are important conditions that must be met. Among these are:

1. Development of Instructional Materials

Textual materials that integrate skills development with problem solving in an articulated fashion are currently almost non-existent. The effort that needs to be made nationally is on the scale of the post-Sputnik curriculum development projects of the 1950s and 1960s. While we must be ready to take advantage of such efforts when and if they do materialize, we cannot afford to wait.

Efforts to produce the materials that will respond to the needs that have been identified will require the cooperative efforts of mathematics educators at the high school and college/university levels and others with curriculum development responsibilities. Input must also be sought from the business and industrial community. Funding levels must be sufficient to allow for the associated projects to extend over several years with accompanying classroom use of the instructional materials that are produced. There must also be provision for the revision of these materials based upon the classroom experience.

2. Teacher (re)Training and Support

It is imperative that school districts, colleges and universities, and, in fact, the total educational community be involved in preparing teachers for classroom implementation of the changes in both content and methodology that are identified in this document. Inservice training must be directed toward assisting teachers in the use of instructional techniques that are based on a problem-solving approach. Funds must be provided for teachers to take applicable college-level courses, to attend conferences, to develop their own instructional materials, and to work on a continuing basis with the colleagues toward effecting the needed improvements in high school mathematics course offerings. Networks need to be established at the various levels (state, ESD, district) to facilitate the sharing of ideas among teachers and to coordinate program evaluation efforts. Professional organizations should also be called upon to play a part through their publications, conferences, and other activities that they sponsor.

3. Instructional Resources

Resource banks must be developed to support teachers in their efforts to implement a unified/articulated study of mathematics effectively under the problem-solving "umbrella." Such banks should provide teachers with sources of information on a wide variety of classroom-tested instructional strategies; use of multiple texts and/or teacher-generated materials; use of manipulatives and consumable materials; use of tutors, community resources, and reference people; use of mathematics labs and library resources; use of calculators; access to computers, software, and data banks of problems and test items; use of audio-visual materials; use of hands-on measurement activities; and use of outside-of-class activities.

Additional resources needed to support instruction include modern copy machines, overhead projectors, and a variety of concrete materials for concept development activities.

B. Integration of Mathematics Topics

It is recommended that the topics from the various areas of mathematics of the high school curriculum be integrated throughout each course sequence. This is the basis of the organization of topics in the course sequence sections of this document.

C. Time Considerations

The time required for implementing an instructional approach that is structured around problem solving will come partially with the adoption of a new attitude toward "proof." It is recommended that the obvious and trivial proofs that occur in present programs should be eliminated. Alternative styles (paragraph, inductive, informal, etc.) should be emphasized with a concurrent reduction in time spent on formal two-column proofs; however, students in the more advanced coursework should be able to justify most of the algebraic and geometric properties with which they come in contact.

It is also recommended that full advantage should be taken of calculators and computers in the performance of routine, repetitive, or otherwise time-consuming tasks for which they are particularly well suited. This will also serve to free additional time for problem-solving pursuits.

D. Graduation Requirements

Current State graduation requirements mandate the successful completion of two years of mathematics. The Agenda for Action of the National Council of Teachers of Mathematics goes beyond this and strongly recommends three years of mathematics for all students in grades 9 through 12. The Mathematics Core Writing Group agrees with this recommendation and encourages school districts throughout the state to develop plans to move toward this goal. It is also critical that the mathematics coursework in which students engage provides for more than just computation skills proficiency. The National Council

of Supervisors of Mathematics position paper on the Ten Basic Skills Areas provides an excellent blueprint for establishing the "core" learnings that should be acquired by all students.

E. Senior Mathematics Requirement

It is recommended that each student at the senior or 12th-grade level be enrolled in a mathematics course and/or career or vocationally oriented course that emphasizes mathematics skills and understandings. Such courses should provide the proximate response toward meeting students' needs to be mathematically "equipped" for entering into the workplace and/or college coursework.

F. Honors Programs

At the present time, high school honors programs in mathematics provide an accelerated course sequence that generally culminates in the study of calculus at the 12th-grade level and provides the possibility of advanced placement for those students who are involved in the program. While the validity of such programs for outstanding mathematics students is recognized, it is recommended that students be enrolled in calculus only after successfully completing the coursework that is detailed in the Four-Year Sequence section that appears on following pages. Further, it is recommended that discussion take place among high school and college/university mathematics educators to determine if there are other courses in addition to calculus that should be considered as the culmination of the honors program.

G. Enrollment of Minorities and Females in Mathematics

Attention must be given to the underrepresentation of females and minorities in the more advanced mathematics coursework. Schools/districts must make concerted efforts toward the recognition of potential among the students in underrepresented groups and recruit them into the advanced courses. Support programs that are designed to help such students achieve success in their mathematics coursework should also be considered. Affiliation with networks and other organizations dedicated to overcoming the obstacles to more balanced enrollment patterns is highly recommended.

H. Teacher Selection

All mathematics courses at the high school level must be taught by teachers who have been trained to teach mathematics and who demonstrate enthusiasm for the subject matter and a concern for their students.

I. Additional Concerns

1. The entire mathematics program at the high school level needs to be supported by accessibility to diagnostic information, effective placement procedures, and reasonable class-size restrictions.

2. Program design must provide for flexibility to allow students to transfer between sequences of study as their aspirations change and capabilities allow.

3. There must be increased coordination of effort among teachers, counselors, and administrators regarding proper placement of students with respect to the special requirements of the coursework of the various sequences of study.

CURRICULUM DEVELOPMENT GUIDELINES

As previously stated, there are three sequences of coursework that are detailed in the curriculum development guidelines that follow. These are:

Four-Year Sequence. This is an integrated course of study that is structured around problem solving, applications, and the acquisition of theory. The sequence is designed for the college-bound student who plans to enter a mathematics-based field of study. Successful completion of the coursework in this sequence will prepare a student for entry into calculus or any other equally demanding culminating coursework in the high school honors program as well as for entry into any first-year college program.

Three-Year Sequence. This is an integrated course of study that is designed for students whose present plans are not directed toward entrance into a mathematics-based college program. Although not as mathematically rigorous as the four-year sequence, students who successfully complete the coursework of this sequence will be prepared (should their plans change) to enter a college-level precalculus program.

Two-Year Sequence. This is an integrated course of study of high school mathematics for the student whose present progress in mathematics, achievement level and aspirations preclude the potential for success if enrolled in either of the other sequences. The amount and level of content that is covered is significantly less than that of the other sequences; however the coursework in this sequence meets the minimum State graduation requirements in mathematics.

The framework of the guidelines for the sequences allows for sufficient flexibility so that students may move from one sequence to another as conditions warrant.

For each of the sequences, the guidelines indicate the suggested years in which topics should be introduced, developed, and mastered. * In general, most topics are introduced during the first year of the sequence and developed in successive years, increasing the level of sophistication with each added exposure. Mastery of some topics is not expected (as indicated by the lack of a number in the "Year Mastered" column).

* Introduce indicates either needed review or the initial exposure to the skills and understandings associated with the topic.

Develop indicates an extension of understanding such that, with teacher guidance, students can apply such understandings to the solution of problems.

Mastery indicates a sufficient understanding of the concepts involved such that the student, without teacher guidance, can apply such understandings to the solution of problems in "new" as well as familiar situations.

Many topics are common to all three sequences with the major differences being reflected in the selection of appropriate activities that provide a match between the student's ability level and the difficulty of the concepts involved. In this regard, it is strongly recommended that:

- Emphasis is placed on hands-on practical experiences in the introduction and development of topics.
- Calculators are used for most computational activities.
- Computers are used whenever appropriate for problem solving and concept development.
- Small-group activities are utilized whenever possible to allow for individual growth through cooperative effort.
- Problem-solving resources are identified and catalogued as they become increasingly available so that the information may be shared with colleagues.

Each section of the guidelines that follow includes an "Instructional Implication" statement. These statements are intended to establish some direction, but not to set limits, on the types of activities and instructional methods that should be employed in the classroom. Curriculum developers should find these suggestions useful in setting the "tone" of the instructional resources that are designed for use in the implementation of a curriculum based upon this document.

I. FOUR-YEAR SEQUENCE

A. CRITICAL THINKING AND PROBLEM SOLVING

STUDENTS SHOULD

		Year Introduced	Year(s) Developed	Year Mastered
1.	Understand and use problem-solving methods. Steps may include:			
	a. formulating the problem;	1	1234	
	b. analyzing and conceptualizing the problem including the use of sketches, physical models, Venn diagrams, and estimation;	1	1234	
	c. identifying appropriate data and tools;	1	1234	
	d. using experimentation and intuition;	1	1234	
	e. transferring skills and strategies to new situations;	1	1234	
	f. drawing on background knowledge;	1	1234	
	g. modeling and predicting outcomes;	1	1234	
	h. finding and interpreting solutions;	1	1234	
	i. checking for reasonableness of results.	1	1234	

Instructional Implications: The need for critical thinking and problem solving does not begin in grade nine. From the earliest grade levels forward, teachers should expose students to situations for which the solution is not immediately obvious. Such problems should require exploration and result in knowledge acquisition. They may have multiple solutions, approximate solutions or both. They may, in fact, have no solution. The teacher should include problems which can be interpreted and approached in various ways. In this context, the teacher may wish to include both packaged computer programs and student-written programs to complement traditional problem-solving methods.

2.	Apply reasoning processes in problem solving that include:			
	a. inductive reasoning;	1	1234	
	b. deductive reasoning;			
	(1) direct and indirect;	1	1234	

(2) symbolic logic;	2	234	
(3) mathematical induction.	2	34	

Instructional Implications: The teacher must become increasingly sensitive to the frustration levels of students, allowing for the development of thinking skills to take place over extended periods of time. Further, teachers should be aware that the open-ended nature of problem solving and the associated thinking skills are such that closure is never reached (accounting for the fact that there are no entries in the "Year mastered" column in this and the preceding section). Teachers should take care to select activities that provide the excitement of discovery that in turn will lead to higher levels of motivation.

3. Understand and use probability and statistical methods in the problem-solving process that include attention to:

a. mean, median, mode;	1	1	1
b. presentation and interpretation of data including the use of graphs, tables, etc.;	1	123	3
c. interpolation and extrapolation;	1	123	4
d. standard deviation;	1	123	3
e. permutations and combinations;	2	23	4
f. probability and odds;	2	234	4
g. sampling techniques.	3	23	

Instructional Implications: Teachers should provide an abundance of "real-life" situations that involve the collection and interpretation of data. Teachers should also capitalize on the potential for the use of calculators and/or computers in facilitating the processing and displaying of data.

B. MEASUREMENT: SKILLS AND UNDERSTANDINGS

STUDENTS SHOULD

1. Understand and apply the factor labeling concept (dimensional analysis): e.g., 50 km/hr x 2 hr = 100 km.

1	12	2

Instructional Implications: Teachers should introduce this concept as the need arises in finding solutions to problems. Activities should be provided that utilize an inductive approach to help students discover the relationships that exist among the various units of measure involved.

2. Use measuring devices in measurement activities involving length, area, volume, capacity, weight (mass), angles, and time.

1	12	2

Instructional Implications: Teachers should provide many opportunities for hands-on measurement activities using such tools as rulers, compasses, protractors, directional compasses, sextants, transits, balance and spring scales, timing devices, etc.

3. Understand the approximate nature of measurement, and such concepts as rounding, greatest possible error, precision, accuracy, relative error, and the use of significant digits in computations involving measurements.

1	1234	

Instructional Implications: Teachers should emphasize a "common-sense" approach to determining the reasonableness of results obtained in computations involving measurements and provide activities that will assist students in becoming adept at guessing and checking, an important problem-solving heuristic. This is also an area in which teachers should utilize applications from other disciplines to motivate the need for the acquisition of the necessary understandings and skills.

4. Know the relationship among units within each system, English and metric; be able to convert measurements in given units to measurements in other units within each system; and be able to use "rule-of-thumb" methods to obtain approximate conversions between the systems (e.g., a meter is a little longer than a yard, etc.).

1	123	3

I-B. continued

Instructional Implications: Teachers should provide hands-on measurement activities that lead students to discover the conversion factors. In particular, it is not sufficient to teach "moving the decimal point" rules for converting among metric units; but instead, instruction should provide students with the means to deduce whether they are to multiply or divide by powers of ten (and by which powers of ten) through experiential activities.

5. Use direct measurement in problem-solving activities such as those involving:

a. visual estimation;	1	1234	
b. similar polygon applications;	1	23	3
c. right triangle applications;	1	23	3
d. use of computer simulations.	3	34	

Instructional Implications: Teachers should provide applications that create the need for the use of problem solving heuristics such as "guess and check," use of either pictorial or physical models, solving similar but simpler problems, etc. Opportunities should be provided for students to experience applications in the world around them: e.g., finding the straight-line distance from one corner of a building to the diagonally opposite corner.

C. GEOMETRY: SKILLS AND UNDERSTANDINGS

STUDENTS SHOULD

1. Understand and use the vocabulary and symbolism associated with geometry.	1	123	3

Instructional Implications: Teachers should provide instruction that will assist students in learning the terminology of geometry so that they will be able to describe properties of geometric figures in terms of basic definitions. Activities should be provided that involve the use of measurement and construction tools, diagrams, graphs, models, etc., to reinforce the learning of vocabulary in meaningful contexts.

I-C. continued

2. Understand and use concepts related to plane and solid geometric figures such as:

a. congruence;	1	123	3
b. similarity;	1	123	3
c. parallelism;	1	123	3
d. perpendicularity;	1	123	3
e. skewness;	1	123	3
f. loci;	1	123	3
g. symmetry;	1	123	3
h. right triangle trigonometry.	1	123	3

Instructional Implications: Teachers should provide experiential activities that lead to generalizations concerning these concepts. In particular, use should be made of physical models with opportunities for students to construct them as well as use models constructed by others. Activities involving applications from the "real world" should be used to develop concepts related to right triangle trigonometry. Opportunities for extensions to other geometries should be afforded.

3. Prove or otherwise justify selected geometric hypotheses.

1	123	

Instructional Implications: Teachers should emphasize alternative styles of justifying hypotheses (paragraph, inductive, intuitive, etc.) with a concurrent reduction in time spent on formal two-column proofs; however, teachers should, at all times, hold students accountable for the methods used and the results obtained. (The time required for implementing an instructional approach that is structured around problem solving may, in part, come with the adoption of a new attitude toward "proof." The obvious and trivial proofs that occur in present programs should be eliminated altogether.)

4. Understand and use the Pythagorean Theorem and elementary trigonometric ratios in solving problems involving right triangles.

1	123	3

Instructional Implications: Teachers should provide opportunities for students to discover the Pythagorean Theorem and right triangle trigonometric ratios through experiential activities that involve construction, measurement and the use of calculators/computers to process the data that is generated by the activities. Applications from the "real world" should be emphasized.

5. Demonstrate ability to perform geometric constructions such as constructing figures that are congruent to given figures, bisecting segments and angles, constructing perpendiculars and lines parallel to given lines, etc.

1	123	3

Instructional Implications: Teachers should provide for an integration of construction activities throughout the study of mathematics, thus facilitating transitions from the concrete to the abstract.

6. Understand and use coordinate geometry concepts in problem solving: e.g., slope, distance, midpoint, equations of lines, conic sections, and transformations.

1	1234	4

Instructional Implications: Teachers should select activities involving applications from business and industry. Extensions should be made to problems requiring the application of linear programming concepts. Extensive use may be made of computer graphics in facilitating the understanding and use of the concepts of coordinate geometry.

7. Understand concepts related to vectors and solve problems that require the geometric addition and subtraction of vectors.

1	1234	4

Instructional Implications: Teachers should help students understand how the language and properties of vectors are used in applied mathematics at this and more advanced levels of mathematics. Applications should be provided that

utilize the visual and kinesthetic power of the geometric vector combined with its algebraic properties to solve problems and construct proofs. Extensions to three dimensions should be made.

8. Understand and use transformational geometry concepts.

2	234	

Instructional Implications: Teachers should provide activities that are drawn from industrial applications, particularly in the area of computer assisted design. The inclusion of transformational geometry concepts in the mathematics curriculum is of importance both from the standpoint of interest and practical application.

9. Understand and use polar coordinates.

3	34	4

Instructional Implications: Teachers should make use of polar graph paper and/or computers in developing understandings and applications associated with polar coordinates.

D. NUMBER PROPERTIES, THEORY AND COMPUTATION

STUDENTS SHOULD

1. Understand and use rules for order of operations and for simplifying expressions in computing with rational numbers, radicals, real numbers, and complex numbers.

1	1234	4

Instructional Implications: Teachers should stress the use of calculators in computing. The use of pencil and paper algorithms should be minimized. Teachers should provide applications including those involving algebraic and geometric concepts (e.g., distance formula, geometric mean, area, volume, etc.) to establish meaningful contexts for the practice and refinement of needed computation skills.

I-D. continued

2. Understand and use set notation and concepts from set theory such as union, intersection, complement, etc.

1	12	2

Instructional Implications: Teachers should provide activities that help students understand the use of sets, set language and set notation in clarifying and unifying algebraic and geometric concepts. The instructional sequence should proceed from the concrete to the abstract.

3. Understand and use the field properties.

1	1234	

Instructional Implications: Teachers should introduce these properties within the contexts of the various number systems. For example, the commutative, associative, and distributive properties and the properties of zero and one should be introduced by means of patterns found by computing with natural numbers and rational numbers. When students exhibit understanding of the properties as they apply to numerical examples, instruction directed toward formal definitions will be meaningful to them. Teachers may then wish to provide activities that extend these understandings to non-numerical systems.

Students should be given numerous opportunities to apply the field properties to problems requiring the simplification of algebraic expressions, solution of equations, and construction of simple algebraic proofs. (Quasi-proof activities are appropriate at this level.)

4. Understand and use exponential and scientific notation.

1	12	2

Instructional Implications: Teachers should provide instruction, as needed, to enable students to generalize place value concepts by expressing the place value of each digit in a numeral as a power of the base (whatever the base). Activities should be provided that involve translating between standard form and exponential notation as well as those that involve translating among various bases. Applications involving scientific notation should be drawn from the physical sciences.

I-D. continued

5. Understand and apply the properties of absolute value.

1	123	4

Instructional Implications: Teachers should provide activities that include the geometric interpretation, algebraic definition, and triangle inequality.

6. Understand the properties of sequences and series and be able to compute scientific terms and sums of arithmetic and geometric sequences.

2	234	4

Instructional Implications: Teachers should utilize activities that involve pattern recognition and the use of patterns in problem-solving activities. Advantage should be taken of the many opportunities for investigation and discovery in this area of mathematics. Use should also be made of non-obvious sequences to illustrate the fallibility of inductive reasoning: e.g., making all possible connections among n points on a circle to obtain the maximum number of regions generates the sequence, 1, 2, 4, 8, 16, 31, 57, . . . and not 1, 2, 4, 8, 16, 32, 64, . . . as expected by most students when given the problem of determining the sequence by connecting successively greater numbers of points. The use of calculators and/or computers should be encouraged in facilitating explorations in this area of mathematics.

7. Understand vectors and vector arithmetic.

2	234	4

Instructional Implications: Teachers should involve students in the use of vectors and vector arithmetic in the investigation of applications from science and industry. Activities should be provided that lead students to understand that this powerful problem-solving tool will become increasingly important as the number and significance of applications continue to increase. This is also an area of mathematics in which the computer will play an increasingly important role.

I-D. continued

8. Understand matrices and computation with matrices.	3	234	4

Instructional Implications: Teachers should make use of computers in helping students achieve the necessary understandings as well as aiding in the related computational processes. Applications should be drawn from business and industry. (It is anticipated that this area of mathematics will assume increasing importance with extensions to new areas of application in the years to come.)

E. ALGEBRA: CONCEPTS AND SKILLS

STUDENTS SHOULD

1. Demonstrate ability to simplify algebraic expressions.	1	1234	4

Instructional Implications: Teachers should provide an abundance of applications to motivate students to acquire necessary algebraic skills: i.e., skill instruction should not be provided in isolation from the applications for which the skills are needed.

2. Solve and graph equations and inequalities including:

a. linear;	1	12	2
b. quadratic;	1	123	3
c. systems in two/three variables;	1	1234	4
d. fractional;	1	123	3
e. radical;	2	23	3
f. absolute value;	2	234	4
g. trigonometric;	2	34	4

h. exponential and logarithmic;	3	34	4
i. applications to linear programming.	3	34	

Instructional Implications: Many of the types of equations/inequalities listed above are those generated by non-trivial; problems in physics, chemistry, business, accounting, communications, game theory, and literally hundreds of other areas. This emphasizes the need for mathematics teachers to maintain communication with teachers in other disciplines. In addition, people from the "outside" community also should be used as resources whenever possible.

Teachers should provide activities in which the use of calculator skills and computer simulations are crucial. Further, teachers should provide opportunities for students to demonstrate their understanding of a problem situation by creating a computer program directed toward the solution of the problem.

When equations involve radicals, the teacher needs to provide activities that lead students to understand both the loss of accuracy and the loss of simplicity that may result when calculators or computers are used.

3. Understand and use ratio and proportion in solving problems including those involving:

a. similarity;	1	123	3
b. geometric mean;	2	23	3
c. direct, inverse, combined and joint variation;	2	23	4
d. probability.	2	23	4

Instructional Implications: Teachers should provide activities that require experimentation (including computer simulations), the use of graphs, scale drawings, models, etc. to obtain and/or display the data needed for solution of problems.

4. Prove or otherwise justify selected algebraic statements.	1	234	

Instructional Implications: Teachers should encourage alternative proof styles (e.g., paragraph, inductive, etc.) with the understanding that "informality" in justifying hypothesis does not excuse students from accountability for the methods used and the results obtained. Teachers should provide for a comparable emphasis on the justification of algebraic properties as is the case for geometric properties.

5. Apply the algebraic manipulation of vectors in problem-solving situations involving:

a. vector addition;	2	234	4
b. scalar and vector multiplication;	2	234	4
c. use of matrices in solving systems of equations.	3	34	4

Instructional Implications: Teachers should provide an abundance of activities that are drawn from applications in business and industry. The computer should be used as an aid in the problem-solving process. Teachers should help students understand that this is an area of mathematics in which new applications are being developed daily.

F. FUNCTIONS

STUDENTS SHOULD

1. Understand the concept of function including:

a. domain and range of a function;	1	12	2
b. distinction between relation and function;	1	12	2
c. graphs of functions.	1	1234	4

Instructional Implications: Teachers should present this strand as a unifying force in mathematics: i.e., many of the topics in higher-level mathematics are developed within the context of "function."

2. Understand and use properties and operations involving functions including:

a. f + g;	2	34	4
b. f - g;	2	34	4
c. f * g;	2	34	4
d. f/g;	2	34	4
e. f(g);	3	34	4
f. inverses;	2	34	4
g. transformations.	3	34	

Instructional Implications: Teachers should provide activities involving applications of functions that develop the need for their understanding and use. Special attention must be given to the process of abstracting functions from problem situations.

3. Use and graph polynomial functions involving ideas such as:

a. symmetry;	2	234	
b. maxima/minima;	3	34	
c. points of inflection;	3	34	
d. theory of equations (including root estimation methods).	2	34	4

Instructional Implications: Whenever possible, teachers should provide activities that involve the use of computers in the development of understandings associated with polynomial functions and their use in the problem-solving process. (Computers are especially useful in displaying the graphs of functions.)

I-F. continued

4.	Use and graph exponential and logarithmic functions.	3	34	4

Instructional Implications: Teachers should place instructional emphasis on logarithms as functions (e.g., to solve growth and decay problems, compound interest problems, etc.) rather than as a computational tool. Teachers should introduce students to the use of non-uniform graph paper e.g., log-log and semi-log) as a means to display graphs or exponential functions.

5.	Understand and apply the concept of limits, derivatives, and continuity.	3	34	

Instructional Implications: Teachers should make use of calculators and computers to introduce the concept of limit at an intuitive level. Activities should be provided in which the derivative is applied to maxima/minima problems and to curve sketching techniques.

6. Understand and use the concept of circular functions to include:

a.	unit circle;	1	234	4
b.	wrapping function;	1	234	4
c.	radian measure;	2	23	3
d.	graphs;	2	34	4
e.	properties of periodic functions, such as period, amplitude, and phase;	2	34	4
f.	trigonometric identities;	3	34	
g.	inverses of functions;	3	34	
h.	non-right triangle applications.	3	34	4

Instructional Implications: Teachers over the several grade levels should develop concepts associated with circular functions by use of a "spiral" approach: i.e., the interdependency of these topics require their early introduction, and a return to them periodically at increasing levels of sophistication.

LIST OF SELECTED RESOURCES

The following listing of instructional resources is designed to provide curriculum developers and classroom teachers with a rich source of ideas and activities that support the goals and philosophy of the GUIDELINES FOR GRADES 9-12 MATHEMATICS CURRICULUM. The listing is not exhaustive; nor will everyone agree that each of the listed resources is of sufficient significance to merit its inclusion. Nevertheless, these are materials that have a number of knowledgeable mathematics educators have identified as eminently useful in their experience and which they believe will be valuable to those involved in the Guidelines implementation tasks that lie ahead.

A. The following publications contain activity pages that may be duplicated for student use and/or activity ideas that may easily be prepared for classroom implementation:

BASIC THINKING SKILLS SERIES (11 Booklets) by Anita Harnadek, 1977, Midwest Publications.

FIGURAL ANALOGIES (series of 3 booklets) by Howard Black and Sandra Black, 1982, Midwest Publications.

FIGURAL CLASSIFICATIONS (series of 3 booklets) by Howard Black and Sandra Black, 1983, Midwest Publications.

FIGURAL SEQUENCES (series of 3 booklets) by Howard Black and Sandra Black, 1982, Midwest Publications.

FIGURAL SIMILARITIES (series of 4 booklets) by Howard Black and Sandra Black, 1983, Midwest Publications.

FINITE DIFFERENCES, A PATTERN-DISCOVERY APPROACH TO PROBLEM SOLVING by Dale Seymour and Margaret Shedd, 1973, Creative Publications.

INDUCTIVE THINKING SKILLS (series of 11 booklets) by Anita Harnadek, 1979, Midwest Publications.

THE MATHEMATICS OF ISLAMIC ART, produced by the Metropolitan Museum of Art, 1979, National Council of Teachers of Mathematics.

MIND BENDERS (series of 11 booklets) by Anita Harnadek, 1978, Midwest Publications.

MULTICULTURAL MATHEMATICS POSTERS AND ACTIVITIES developed by the Mathematics Office, Seattle Public Schools, 1980, National Council of Teachers of Mathematics.

PROBLEM SOLVING, A HANDBOOK FOR TEACHERS by Stephen Krulik and Jesse A. Rudnick, 1980, Allyn and Bacon.

PROBLEM SOLVING IN MATHEMATICS, GRADE 9 ALGEBRA, Lane County Mathematics Project, 1983, Dale Seymour Publications.

PROBLEMS IN ANALYTIC GEOMETRY by R. J. Palmaccio, 1984, J. Weston Walch.

PUZZLES IN SPACE by David Stonerod, 1982, Creative Publications.

SCI-MATH, APPLICATIONS IN PROPORTIONAL PROBLEM SOLVING, MODULE ONE and MODULE TWO by Madeline P. Goostein, 1983, Addison Wesley.

A SOURCEBOOK FOR TEACHING PROBLEM SOLVING by Stephen Krulik and Jesse A. Rudnick, 1984, Allyn and Bacon.

SPACES (SOLVING PROBLEMS OF ACCESS TO CAREERS IN ENGINEERING AND SCIENCE), developed by The Lawrence Hall of Science, 1982, Dale Seymour Publications.

B. The following publications are appropriate for direct use by students and/or may serve as idea "banks" for curriculum developers as well as classroom teachers:

AHA! GOTCHA, PARADOXES TO PUZZLE AND DELIGHT by Martin Gardner, 1982, W. H. Freeman.

AHA! INSIGHT by Martin Gardner, 1978, Scientific American, W.H. Freeman.

ALGEBRA THROUGH APPLICATIONS, PART 1 and PART 2 by Zalman Usiskin, 1979, National Council of Teachers of Mathematics.

CREATING ESCHER-TYPE DRAWINGS by E. R. Ranucci and J. L. Teeters, 1977, Creative Publications.

CREATIVE CONSTRUCTIONS by Dale Seymour and Reuben Schadler, 1974, Creative Publications.

CRITICAL THINKING, BOOKS 1 and 2, by Anita Harnadek, 1980, Midwest Publications.

THE DIVINE PROPORTION by H. E. Huntley, 1970, Dover Publications.

THE DOME BUILDERS HANDBOOK by John Prenis, 1973, Philadelphia Running Press.

THE DOT AND THE LINE by Norton Juster, 1963, Random House.

THE EDUCATION OF T. C. MITTS by Hugh O. Lieber and Lillian R. Lieber, 1978, W. W. Norton.

FANTASIA MATHEMATICA by Clifton Fadiman, 1958, Simon and Schuster.

FLATLAND by Edwin A. Abbott, 1952, Dover Publications.

GEODESIC MATH AND HOW TO USE IT by Hugh Kenner, 1976, University of California Press.

THE GEOMETRY OF ART AND LIFE by Mitila Ghyka, 1978, William Gannon.

THE GRAPHIC WORK OF M. C. ESCHER by M. C. Escher, 1977, Ballantine Books.

HOW TO LIE WITH STATISTICS by Darrell Huff and Irving Geis, 1954, W. W. Norton.

HUMAN VALUES AND SCIENCE, ART AND MATHEMATICS by Hugh O. Lieber and Lillian R. Lieber, 1961, W. W. Norton.

THE IMPOSSIBLE IN MATHEMATICS by Irving Adler, 1957, National Council of Teachers of Mathematics.

MAGIC CUBES - NEW RECREATIONS by William H. Benson and Oswald Jacoby, 1981, Dover Publications.

MAGIC SQUARES AND CUBES by W. S. Andrews, 1917, Dover Publications.

MATHEMATICAL BAFFLERS, edited by Angela Dunn, 1980, Dover Publications.

MATHEMATICAL FUN, GAMES AND PUZZLES by Jack Firohlichstein, 1967, Dover Publications.

MATHEMATICAL GEMS by I. Ross Honsberger, 1979, Mathematical Association of America.

THE MATHEMATICAL MAGPIE by Clifton Fadiman, 1962, Simon and Schuster.

MATHEMATICAL MODELS by Henry M. Cundy and A. D. Rollett, 1961, Oxford University Press.

MATHEMATICS, A HUMAN ENDEAVOR by Harold R. Jacobs, 1982, W. H. Freeman.

MATHEMATICS MAGIC AND MYSTERY by Martin Gardner, 1956, Dover Publications.

MATHEMATICS ON VACATION by Joseph S. Madachy, 1966, Charles Scribner's Sons.

THE MOSCOW PUZZLES by Boris A. Kordensky, 1972, Creative Publications.

NEW RECREATIONS WITH MAGIC SQUARES by William H. Benson and Oswald Jacoby, 1976, Dover Publications.

100 GEOMETRIC GAMES by Pierre Berloquin, 1979, Charles Scribner's Sons.

101 PUZZLE PROBLEMS by Nathaniel B. Bates and Sanderson M. Smith, 1980, Bates Publishing Co.

101 PUZZLES IN THOUGHT AND LOGIC by C. R. Wylie Jr., 1957, Dover Publications.

PATTERNS IN SPACE by Robert Beard, 1973, Creative Publications.

POLYHEDRA PRIMER by Peter Pearce and Susan Pearce, 1978, Dale Seymour Publications.

POLYHEDRON MODELS FOR THE CLASSROOM by Magnus J. Wenninger, 1966, National Council of Teachers of Mathematics.

THE POWER OF LIMITS, PROPORTIONAL HARMONICS IN NATURE, ART, AND ARCHITECTURE by Gyorgy Doczi, 1981, Shambhala Publications.

PUZZLE THINKING by Franeete Walberg, 1980, The Franklin Institute Press.

READING, WRITING, AND DOING MATHEMATICAL PROOFS
GEOMETRY
ADVANCED MATHEMATICS
by Daniel Solow, 1984, Dale Seymour Publications.

RECREATIONAL PROBLEMS IN GEOMETRIC DISSECTIONS AND HOW TO SOLVE THEM by Harry Lindgren, 1972, Dover Publications.

SCIENCE FICTION PUZZLE TALES by Martin Gardner, 1981, Crown Publishers.

SCIENTIFIC AMERICAN BOOK OF MATHEMATICAL PUZZLES AND DIVERSIONS by Martin Gardner, 1963, Simon and Schuster.

SECOND BOOK OF MATHEMATICAL BAFFLERS, edited by Anglea Dunn, 1983, Dover Publications.

THE 2ND SCIENTIFIC AMERICAN BOOK OF MATHEMATICAL PUZZLES AND DIVERSIONS by Martin Gardner, 1965, Simon and Schuster.

SOME "PRIME" COMPARISONS by Stephen I. Brown, 1978, National Council of Teachers of Mathematics.

SPHERELAND by Dionys Burger, 1965, Thomas Y. Crowell.

THE UNEXPECTED HANGING AND OTHER MATHEMATICAL DIVERSIONS by Martin Gardner, 1969, Simon and Schuster.

WHOM THE GODS LOVE: THE STORY OF EVARISTE GALOIS by Leopold Infeld, 1948, National Council of Teachers of Mathematics.

WOMEN IN MATHEMATICS by Lynn M. Osen, 1974, The MIT Press.

C. The following publications provide teachers with rich sources of problems and activity ideas for use with their students:

APPLICATIONS OF SECONDARY SCHOOL MATHEMATICS by Bernice Kastner, 1978, National Council of Teachers of Mathematics.

THE CONTEST PROBLEMS BOOKS II, III, AND IV from the Annual High School Mathematics Examinations, 1961-1982, National Council of Teachers of Mathematics.

EXPERIENCES IN VISUAL THINKING, 2ND ED. by Robert McKim, 1980, Brooks/Cole.

GREAT MOMENTS IN MATHEMATICS AFTER 1650 by Howard W. Eves, 1982, Mathematical Association of America.

GREAT MOMENTS IN MATHEMATICS BEFORE 1650 by Howard W. Eves, 1982, Mathematical Association of America.

GEOMETRY: AN INVESTIGATIVE APPROACH by Phares G. O'Daffer and Stanley R. Clemens, 1977, Addison Wesley.

HISTORICAL TOPICS IN ALGEBRA, part of the 1971 Yearbook, National Council of Teachers of Mathematics.

IN MATHEMATICAL CIRCLES (2 Volumes) by Howard W. Eves, 1969, Prindle, Weber and Schmidt.

INTERNATIONAL MATHEMATICAL OLYMPIADS 1959-1977, compiled by Samuel L. Greitzer, 1979, National Council of Teachers of Mathematics.

INTRODUCTION TO NON-EUCLIDEAN GEOMETRY by Harold Wolfe, 1945, Irvington.

AN INTRODUCTION TO THE HISTORY OF MATHEMATICS by Howard W. Eves, 1983, Saunders College Publishing.

MATHEMATICAL CHALLENGES II - PLUS SIX, edited by Thomas Hill, 1974, National Council of Teachers of Mathematics.

MATHEMATICAL CIRCLES ADIEU by Howard W. Eves, 1977, Prindle, Weber and Schmidt.

MATHEMATICAL CIRCLES REVISITED by Howard W. Eves, 1971, Prindle, Weber and Schmidt.

MATHEMATICAL CIRCLES SQUARED by Howard W. Eves, 1972, Prindle, Weber and Schmidt.

MATHEMATICAL IDEAS by Charles D. Miller and Vern E. Heeron, 1980, Scott, Foresman.

MATHEMATICS CONTEST PROBLEMS FOR JUNIOR AND SENIOR HIGH SCHOOL, Saint Mary's College/Brother Alfred Brousseau, 1971, Creative Publications.

MATHEMATICS - PROBLEM SOLVING THROUGH RECREATIONAL MATHEMATICS by Bonnie Averbach and Orin Chein, 1980, W. H. Freeman.

MATHEMATICS TEACHER, published nine times yearly by the National Council of Teachers of Mathematics.

MATHEMATICS, THE MAN-MADE UNIVERSE, by Sherman K. Stein, 1969, W. H. Freeman.

THE NATURE OF MATHEMATICS by Karl J. Smith, 1984, Brooks/Cole Publishing Co.

NUMBERS AND INFINITY by Sondheimer and Rogerson, 1981, Cambridge University Press.

POLYHEDRAL MODELS by Magnus Wenninger, 1971, Cambridge University Press.

PROBLEM SOLVING AND COMPREHENSION, A SHORT COURSE IN ANALYTICAL REASONING by Arthur Whimbey and Jack Lochhead, 1980, The Franklin Institute Press.

RECREATIONS IN THE THEORY OF NUMBERS by Albert H. Beiler, 1963, Dover Publications.

SIXTY CHALLENGING PROBLEMS WITH BASIC SOLUTIONS by Donald Spencer, 1979, Hayden Book Co.

A SOURCEBOOK OF APPLICATIONS OF SCHOOL MATHEMATICS, prepared by Donald Bushaw and others, 1980, National Council of Teachers of Mathematics.

SPHERICAL MODELS by Magnus Wenninger, 1979, Cambridge University Press.

STATISTICS BY EXAMPLE by Frederick Mosteller et al, 1976, Addison Wesley.

TEACHING MATHEMATICS, A SOURCEBOOK OF AIDS, ACTIVITIES, AND STRATEGIES by Max A. Sobel and Evan M. Maletsky, 1975, Prentice-Hall.

TEACHING MATHEMATICS IN THE SECONDARY SCHOOL by Alfred S. Posamentier and Jay Stepelman, 1981, Charles E. Merrill.

TEACHING STATISTICS AND PROBABILITY, 1981 Yearbook, National Council of Teachers of Mathematics.

D. Other materials of general interest to both curriculum developers and classroom teachers are listed below:

AN AGENDA FOR ACTION - RECOMMENDATIONS FOR SCHOOL MATHEMATICS OF THE 1980s, 1980, National Council of Teachers of Mathematics.

THE AGENDA IN ACTION, 1983 Yearbook of the National Council of Teachers of Mathematics.

APPLICATIONS IN SCHOOL MATHEMATICS, 1979 Yearbook of the National Council of Teachers of Mathematics.

CLASSROOM IDEAS FROM RESEARCH ON SECONDARY SCHOOL MATHEMATICS by Donald Dessard and Marilyn Suydam, 1983, National Council of Teachers of Mathematics.

COMPUTERS IN MATHEMATICS, edited by David H. Ahl, 1979, Creative Computing Press.

COMPUTERS IN MATHEMATICS EDUCATION, 1984 Yearbook of the National Council of Teachers of Mathematics.

COMPUTERS IN TEACHING MATHEMATICS by P. Kelman et al, 1983, Addison Wesley.

COMPUTING AND MATHEMATICS: THE IMPACT ON SECONDARY SCHOOL MATHEMATICS, edited by James Fey, 1984, National Council of Teachers of Mathematics.

CONCEPTUAL BLOCKBUSTING by James L. Adams, 1980, W. W. Norton.

GUIDELINES FOR K-8 MATHEMATICS CURRICULUM, published by the Office of the Superintendent of Public Instruction, Olympia, WA, 1984.

THE HIGH SCHOOL MATHEMATICS LIBRARY by William L. Schaaf, 1982, National Council of Teachers of Mathematics.

HM MATH STUDY SKILLS PROGRAM by Catherine D. Tobin, 1980, National Council of Teachers of Mathematics.

HOW TO SOLVE IT by George Polya, 1957, Princeton University Press.

INDUCTION AND ANALOGY IN MATHEMATICS by George Polya, 1954, Princeton University Press.

MATH EQUALS by Terri Perl, 1978, Addison Wesley.

MATHEMATICAL DISCOVERY by George Polya, 1962, John Wiley and Sons.

MATHEMATICS: AN INTRODUCTION TO ITS SPIRIT AND USE, Readings from the Scientific American, 1979, W. H. Freeman.

MATHEMATICS PROJECTS HANDBOOK by Adrien L. Hess, 1982, National Council of Teachers of Mathematics.

PATTERNS OF PLAUSIBLE INFERENCE by George Polya, 1969, University of Princeton Press.

PROBLEM SOLVING IN SCHOOL MATHEMATICS, 1980 Yearbook of the National Council of Teachers of Mathematics.

SCHOOL MATHEMATICS: OPTIONS FOR THE 1990s, Volume I - Chairman's Report of a Conference cosponsored by the U.S. Department of Education, the National Council of Teachers of Mathematics and the Wisconsin Center for Education Research.

THE 10 BASIC SKILLS IN MATHEMATICS, Position Paper of the National Council of Supervisors of Mathematics, 1977.

10 ESSENTIAL IDEAS FOR BETTER TEACHING by Marilyn Suydam, Spring 1984 Special Edition of Washington Mathematics, Washington State Mathematics Council.

THINKING MATHEMATICALLY by John Mason et al, 1982, Addison Wesley.

USE EQUALS TO PROMOTE THE PARTICIPATION OF WOMEN IN MATHEMATICS by Alice Kaseberg, Nancy Kreinberg and Diane Downie, 1980, Lawrence Hall of Science, University of California.

WHAT SHOULD WE TEACH AS GEOMETRY? by Branko Grunbaum, Winter 1984-1985 Special Edition of Washington Mathematics, Washington State Mathematics Council.

II. THREE-YEAR SEQUENCE

A. CRITICAL THINKING AND PROBLEM SOLVING

STUDENTS SHOULD

	Year Introduced	Year(s) Developed	Year Mastered
1. Understand and use problem-solving methods. Steps may include:			
a. formulating the problem;	1	23	
b. analyzing and conceptualizing the problem including the use of sketches, physical models, Venn diagrams, and estimation;	1	23	
c. identifying appropriate data and tools;	1	23	
d. using experimentation and intuition;	1	23	
e. transferring skills and strategies to new situations;	1	23	
f. drawing on background knowledge;	1	23	
g. modeling and predicting outcomes;	1	23	
h. finding and interpreting solutions;	1	23	
i. checking for reasonableness of results.	1	23	

Instructional Implications: From the earliest grade levels forward, teachers should expose students to situations for which the solution is not immediately obvious. Such problems should require exploration and result in knowledge acquisition. They may have multiple solutions, approximate solutions or both. They may, in fact have no solution. The teacher should include problems which can be interpreted and approached in various ways. In this context, the teacher may wish to include both packaged computer programs and student-written programs to complement traditional problem-solving methods.

2. Apply reasoning processes in problem solving that include:			
a. inductive reasoning;	1	23	
b. both direct and indirect deductive reasoning.	1	23	

II-A. continued

Instructional Implications: The teacher must become increasingly sensitive to the frustration levels of students, allowing for the development of thinking skills to take place over extended periods of time. Further, teachers should be aware that the open-ended nature of problem solving and the associated thinking skills are such that closure is never reached. Teachers should take care to select activities that provide the excitement of discovery which in turn will lead to higher levels of motivation.

3. Understand and use probability and statistical methods in the problem-solving process that include attention to:

a. mean, median, mode;	1	1	2
b. presentation and interpretation of data including the use of graphs, tables, etc.;	1	12	3
c. interpolation and extrapolation;	2	23	
d. permutations and combinations;	2	23	
e. probability and odds;	2	23	
f. sampling techniques.	2	23	

Instructional Implications: Teachers should provide an abundance of "real-life" situations that involve the collection and interpolation of data. Teachers should also capitalize on the potential for the use of calculators and/or computers in facilitating the processing and displaying of data.

B. MEASUREMENT: SKILLS AND UNDERSTANDING

STUDENTS SHOULD

1. Understand and apply the factor labeling concept (dimensional analysis): e.g., 50 km/hr x 2 hr is 100 km, etc.	1	12	3

Instructional Implications: Teachers should introduce this concept as the need arises in finding solutions to problems. Teachers should provide activities that utilize an inductive approach to help students discover the relationships that exist between the various units of measure involved.

2. Use measuring devices in measurement activities involving length, area, volume, capacity, weight (mass), angles, and time.

1	12	3

Instructional Implications: Teachers should provide numerous opportunities for students to engage in hands-on measurement activities using such tools as rulers, compasses, protractors, directional compasses, sextants, transits, balance and spring scales, timing devices, etc.

3. Understand the approximate nature of measurement, rounding, and the use of significant digits in computations involving measurements.

1	123	

Instructional Implications: Teachers should provide direct measurement activities that involve estimation, rounding, comparison, and the use of significant digits to solve problems (e.g., perimeter, area, volume).

4. Know the relationship among units within each system, English and metric; be able to convert measurements in given units to measurements in other units within each system; and be able to use "rule-of-thumb" methods to obtain approximate conversions between the systems (e.g., a meter is a little longer than a yard, etc.).

1	123	3

Instructional Implications: Teachers should provide hands-on measurement activities that lead students to discover the conversion factors. In particular, it is not sufficient to teach "moving the decimal point" rules for converting among metric measurements; but instead, students should deduce whether they are to multiply or divide by powers of ten (and by which powers of ten) through experiential activities.

5. Use indirect measurement in problem-solving activities such as those involving:

a. visual estimation;	1	123	
b. similar polygon applications;	2	23	
c. right triangle applications.	2	23	

II-B. continued

Instructional Implications: Teachers should provide situations requiring the use of problem solving heuristics such as "guess and check," use of either pictorial or physical models, solving similar but simpler problems, etc. Opportunities should be provided for students to experience applications in the world around them: e.g., finding the straight-line distance from one corner of a building to the diagonally opposite corner.

C. GEOMETRY: SKILLS AND UNDERSTANDINGS

STUDENTS SHOULD

1. Understand and use the vocabulary and symbolism associated with geometry.	1	23	

Instructional Implications: Teachers should provide instruction that will assist students in learning the terminology of geometry so that they will be able to describe properties of geometric figures in terms of basic definitions. Activities should be provided that involve the use of measurement and construction tools, diagrams, graphs, models, etc., to reinforce the learning of vocabulary in meaningful contexts.

2. Understand and use concepts related to plane and solid geometric figures such as:

a. congruence;	1	23	
b. similarity;	1	23	
c. parallelism;	1	23	
d. perpendicularity;	1	23	
e. skewness;	1	23	
f. symmetry;	1	23	
g. circles.	1	23	

Instructional Implications: Teachers should make extensive use of physical models and present as many "real world" examples of each concept as possible. Students should be encouraged to look for such examples within their environment. They should also be given opportunities to construct their own models to illustrate the various concepts.

3. Understand and use the Pythagorean Theorem in solving problems involving right triangles.	1	1	2

Instructional Implications: Teachers should provide opportunities for students to discover the Pythagorean Theorem through experiential activities that involve construction, measurement and the use of calculators/computers to process the data that is generated by the activities. Applications from the "real world" should be emphasized.

4. Demonstrate ability to perform geometric constructions such as constructing figures that are congruent to given figures, bisecting segments and angles, constructing perpendiculars and lines parallel to given lines, etc.	1	23	

Instructional Implications: Teachers should provide for an integration of construction activities throughout the study of mathematics, thus facilitating transitions from the concrete to the abstract.

5. Understand and use coordinate geometry concepts in problem solving: e.g., slope, distance, midpoint, equations of lines, conic sections.	1	23	

Instructional Implications: Teachers should select activities involving applications from business and industry. Extensive use may be made of computer graphics in facilitating the understanding and use of the concepts of coordinate geometry.

6. Understand and use transformational geometry concepts.	1	23	

Instructional Implications: Teachers should provide activities that relate to industrial applications, particularly in the area of computer-assisted design. The inclusion of transformational geometry concepts in the mathematics curriculum is of importance both from the standpoints of interest and practical application.

7. Justify selected geometric hypotheses.

2	23	

Instructional Implications: Teachers should emphasize alternative styles of justifying hypotheses (paragraph, inductive, intuitive, etc.) with a concurrent reduction in time spent on formal two-column proofs; however, teachers should, at all times, hold students accountable for the methods used and the results obtained. (The time required for implementing an instructional approach that is structured around problem solving may, in part, come with the adoption of a new attitude toward "proof." The obvious and trivial proofs that occur in present programs should be eliminated altogether.)

8. Understand and use trigonometric ratios in solving problems involving right triangles.

2	3	

Instructional Implications: Teachers should provide opportunities for students to discover right triangle trigonometric ratios through experiential activities that involve construction, measurement and the use of calculators/computers to process the data that is generated by the activities. Applications from the "real world" should be emphasized.

9. Understand and use elementary concepts of polar coordinates and vectors.

2	3	

Instructional Implications: Teachers should provide students with simple applications involving polar coordinates and vectors. The use of computer graphics in clarifying the associated concepts should be exploited.

D. NUMBER PROPERTIES: THEORY AND COMPUTATION

STUDENTS SHOULD

1. Understand and use rules for order of operations and for simplifying expressions in computing with rational numbers, radicals, and real numbers.

1	123	3

Instructional Implications: Teachers should stress the use of calculators in computing. The use of pencil and paper algorithms should be minimized. Teachers should provide applications including those involving algebraic and geometric concepts (e.g., distance formula, geometric mean, area, volume, etc.) to establish meaningful contexts for the acquisition of the needed computation skills.

2. Understand and use set notation and concepts from set theory such as union, intersection, complement, etc.

1	12	2

Instructional Implications: Teachers should provide activities that help students understand the use of sets, set language and set notation in clarifying and unifying algebraic and geometric concepts. The instructional sequence should proceed from the concrete to the abstract.

3. Be able to use properties of real numbers.

1	123	3

Instructional Implications: Teachers should introduce these properties within the contexts of the various number systems. For example, the commutative, associative, and distributive properties and the properties of zero and one should be introduced by means of patterns found by computing with natural numbers and rational numbers. When students exhibit understanding of the properties as they apply to numerical examples, instruction directed toward formal definitions will be meaningful to them.

Students should be given numerous opportunities to apply the field properties to problems requiring the simplification of algebraic expressions, solution of equations, and construction of simple algebraic proofs. (Quasi-proof activities are appropriate at this level.)

4. Understand and use exponential and scientific notation.

1	123	3

Instructional Implications: Teachers should provide instruction, as needed, to enable students to generalize place value concepts by expressing the place value of each digit in a numeral as a power of the base (whatever the base). Activities should be provided that involve

translating between standard form and exponential notation as well as those that involve translating among various bases. Applications involving scientific notation should be drawn from the physical sciences.

5. Be able to compute a specific term or the sum of a finite number of terms of both arithmetic and geometric sequences.

1	123	3

Instructional Implications: Teachers should utilize activities that involve pattern recognition and the use of patterns in problem-solving activities. Advantage should be taken of the many opportunities for investigation and discovery in this area of mathematics. Use should also be made of non-obvious sequences to illustrate the fallibility of inductive reasoning: e.g., making all possible connections among n points on a circle to obtain the maximum number of regions generates the sequence, 1, 2, 4, 8, 16, 31, 57, . . . and not 1, 2, 4, 8, 16, 32, 64, . . . as expected by most students when they are given the problem of determining the sequence of the number of regions by connecting successively greater numbers of points. The use of calculators and/or computers should be encouraged in facilitating explorations in this area of mathematics.

E. ALGEBRA: CONCEPTS AND SKILLS

STUDENTS SHOULD

1. Demonstrate the ability to simplify algebraic expressions.

1	123	3

Instructional Implications: Teachers should provide an abundance of applications to motivate the acquisition of algebraic skills: i.e., skills instruction should not be provided in isolation from the applications for which the skills are needed.

2. Solve and graph sentences, including:

a. linear equations, inequalities;	1	12	3
b. quadratic equations;	2	23	
c. systems of linear equations and inequalities in two variables;	2	23	

d. fractional equations;	2	23	
e. simple radical equations;	2	23	
f. simple equations involving absolute value.	2	23	

Instructional Implications: Many of the types of equations/inequalities listed above are those generated by non-trivial problems in physics, chemistry, business, accounting, communications, game theory, and literally hundreds of other areas. This emphasizes the need for mathematics teachers to maintain communication with teachers in other disciplines. In addition, people from the "outside" community should be used as resources whenever possible.

Teachers should provide activities in which the use of calculator skills and computer simulations are crucial. Further, teachers should provide opportunities for students to demonstrate their understanding of a problem situation by creating a computer program directed toward the solution of the problem.

When equations involve radicals, the teacher needs to provide activities that lead students to understand both the loss of accuracy and the loss of simplicity that may result when calculators or computers are used.

3. Understand and use ratio and proportion in solving problems including those involving:

a. similarity;	2	23	3
b. direct and inverse variation;	2	23	
c. probability.	3	3	

Instructional Implications: Teachers should provide activities that require experimentation (including computer simulations), the use of graphs, scale drawings, models, etc. to obtain and/or display the data needed for the solution of problems.

F. FUNCTIONS

STUDENTS SHOULD

1. Understand the concept of function including:

a.	domain and range of a function;	1	123	3
b.	distinction between relation and function;	1	123	3
c.	graphs of functions;	1	23	
d.	sum and difference of functions.	3	3	

Instructional Implications: Teachers should develop these concepts by using concrete and representational illustrations and demonstrations such as a "function machine." Activities should be provided that allow students to become familiar with many different kinds of functions. The use of sequences and patterns should be emphasized. Opportunities should be provided for students to represent functions by formulas, tables, and graphs.

2. Use and graph exponential and logarithmic functions.

3	3	

Instructional Implications: Teachers should provide instructional emphasis on logarithms as functions (e.g., as applied to the solution of growth and decay problems, compound interest problems, etc.) rather than as a computational tool. Teachers should introduce students to the use of non-uniform graph paper (e.g., log-log and semi-log) as a means to display graphs of exponential functions.

LIST OF SELECTED RESOURCES

The following listing of instructional resources is designed to provide curriculum developers and classroom teachers with a rich source of ideas and activities that support the goals and philosophy of the GUIDELINES FOR GRADES 9-12 MATHEMATICS CURRICULUM. The listing is not exhaustive; nor will everyone agree that each of the listed resources is of sufficient significance to merit its inclusion. Nevertheless, these are materials that have a number of knowledgeable mathematics educators have identified as eminently useful in their experience and which they believe will be valuable to those involved in the Guidelines implementation tasks that lie ahead.

A. The following publications contain activity pages that may be duplicated for student use and/or activity ideas that may easily be prepared for classroom implementation:

ART "N" MATH by Karen Billings, Carol Campbell, and Alice Schwandt, 1975, Action Math Associates.

BASIC THINKING SKILLS SERIES (11 Booklets) by Anita Harnadek, 1977, Midwest Publications.

FIGURAL ANALOGIES (series of 3 booklets) by Howard Black and Sandra Black, 1982, Midwest Publications.

FIGURAL CLASSIFICATIONS (series of 3 booklets) by Howard Black and Sandra Black, 1983, Midwest Publications.

FIGURAL SEQUENCES (series of 3 booklets) by Howard Black and Sandra Black, 1982, Midwest Publications.

FIGURAL SIMILARITIES (series of 4 booklets) by Howard Black and Sandra Black, 1983, Midwest Publications.

FINITE DIFFERENCES, A PATTERN-DISCOVERY APPROACH TO PROBLEM SOLVING by Dale Seymour and Margaret Shedd, 1973, Creative Publications.

INDUCTIVE THINKING SKILLS (series of 11 booklets) by Anita Harnadek, 1979, Midwest Publications.

THE MATHEMATICS OF ISLAMIC ART, produced by the Metropolitan Museum of Art, 1979, National Council of Teachers of Mathematics.

MIND BENDERS (series of 11 booklets) by Anita Harnadek, 1978, Midwest Publications.

MULTICULTURAL MATHEMATICS POSTERS AND ACTIVITIES developed by the Mathematics Office, Seattle Public Schools, 1980, National Council of Teachers of Mathematics.

PROBLEM SOLVING, A HANDBOOK FOR TEACHERS by Stephen Krulik and Jesse A. Rudnick, 1980, Allyn and Bacon.

PROBLEM SOLVING IN MATHEMATICS, GRADE 9 ALGEBRA, Lane County Mathematics Project, 1983, Dale Seymour Publications.

PROBLEMS IN ANALYTIC GEOMETRY by R. J. Palmaccio, 1984, J. Weston Walch.

PUZZLES IN SPACE by David Stonerod, 1982, Creative Publications.

SCI-MATH, APPLICATIONS IN PROPORTIONAL PROBLEM SOLVING, MODULE ONE and MODULE TWO by Madeline P. Goostein, 1983, Addison Wesley.

A SOURCEBOOK FOR TEACHING PROBLEM SOLVING by Stephen Krulik and Jesse A. Rudnick, 1984, Allyn and Bacon.

SPACES (SOLVING PROBLEMS OF ACCESS TO CAREERS IN ENGINEERING AND SCIENCE), developed by The Lawrence Hall of Science, 1982, Dale Seymour Publications.

SPIRIT MASTERS ON CALCULATORS IN GEOMETRY AND MEASUREMENT by Charles H. Lund, 1981, J. Westen Walch.

USING CALCULATORS TO DEVELOP THOUGHTFUL BEHAVIOR IN MATHEMATICS by Edward C. Beardslee, 1984, Washington State Mathematics Council.

WHEN ARE WE EVER GONNA HAVE TO USE THIS? by Hal Saunders, 1981, Dale Seymour Publications.

B. The following publications are appropriate for direct use by students and/or may serve as idea "banks" for curriculum developers as well as classroom teachers:

AHA! GOTCHA, PARADOXES TO PUZZLE AND DELIGHT by Martin Gardner, 1982, W. H. Freeman.

AHA! INSIGHT by Martin Gardner, 1978, Scientific American, W.H. Freeman.

ALGEBRA THROUGH APPLICATIONS, PART 1 and PART 2 by Zalman Usiskin, 1979, National Council of Teachers of Mathematics.

CREATING ESCHER-TYPE DRAWINGS by E. R. Ranucci and J. L. Teeters, 1977, Creative Publications.

CREATIVE CONSTRUCTIONS by Dale Seymour and Reuben Schadler, 1974, Creative Publications.

CRITICAL THINKING, BOOKS 1 and 2, by Anita Harnadek, 1980, Midwest Publications.

THE DOME BUILDERS HANDBOOK by John Prenis, 1973, Philadelphia Running Press.

THE DOT AND THE LINE by Norton Juster, 1963, Random House.

THE EDUCATION OF T. C. MITTS by Hugh O. Lieber and Lillian R. Lieber, 1978, W. W. Norton.

EUREKA by Dale Seymour and Richard Gidley, 1968, Creative Publications.

FANTASIA MATHEMATICA by Clifton Fadiman, 1958, Simon and Schuster.

GEODESIC MATH AND HOW TO USE IT by Hugh Kenner, 1976, University of California Press.

THE GEOMETRY OF ART AND LIFE by Mitila Ghyka, 1978, William Gannon.

THE GRAPHIC WORK OF M. C. ESCHER by M. C. Escher, 1977, Ballantine Books.

HOW TO LIE WITH STATISTICS by Darrell Huff and Irving Geis, 1954, W. W. Norton.

HUMAN VALUES AND SCIENCE, ART AND MATHEMATICS by Hugh O. Lieber and Lillian R. Lieber, 1961, W. W. Norton.

MAGIC SQUARES AND CUBES by W. S. Andrews, 1917, Dover Publications.

MATHEMATICAL FUN, GAMES AND PUZZLES by Jack Firohlichstein, 1967, Dover Publications.

THE MATHEMATICAL MAGPIE by Clifton Fadiman, 1962, Simon and Schuster.

MATHEMATICAL MODELS by Henry M. Cundy and A. D. Rollett, 1961, Oxford University Press.

MATHEMATICS, A HUMAN ENDEAVOR by Harold R. Jacobs, 1982, W. H. Freeman.

NEW RECREATIONS WITH MAGIC SQUARES by William H. Benson and Oswald Jacoby, 1976, Dover Publications.

100 GEOMETRIC GAMES by Pierre Berloquin, 1979, Charles Scribner's Sons.

150 PUZZLES IN CRYPT-ARITHMETIC by Maxey Brooke, 1969, Dover Publications.

101 PUZZLE PROBLEMS by Nathaniel B. Bates and Sanderson M. Smith, 1980, Bates Publishing Co.

101 PUZZLES IN THOUGHT AND LOGIC by C. R. Wylie Jr., 1957, Dover Publications.

PATTERNS IN SPACE by Robert Beard, 1973, Creative Publications.

POLYHEDRA PRIMER by Peter Pearce and Susan Pearce, 1978, Dale Seymour Publications.

POLYHEDRON MODELS FOR THE CLASSROOM by Magnus J. Wenninger, 1966, National Council of Teachers of Mathematics.

PUZZLE THINKING by Franeete Walberg, 1980, The Franklin Institute Press.

READING, WRITING, AND DOING MATHEMATICAL PROOFS
GEOMETRY
ADVANCED MATHEMATICS
by Daniel Solow, 1984, Dale Seymour Publications.

RECREATIONAL PROBLEMS IN GEOMETRIC DISSECTIONS AND HOW TO SOLVE THEM by Harry Lindgren, 1972, Dover Publications.

SCIENCE FICTION PUZZLE TALES by Martin Gardner, 1981, Crown Publishers.

SOME "PRIME" COMPARISONS by Stephen I. Brown, 1978, National Council of Teachers of Mathematics.

WOMEN IN MATHEMATICS by Lynn M. Osen, 1974, The MIT Press.

C. The following publications provide teachers with rich sources of problems and activity ideas for use with their students:

APPLICATIONS OF SECONDARY SCHOOL MATHEMATICS by Bernice Kastner, 1978, National Council of Teachers of Mathematics.

EXPERIENCES IN VISUAL THINKING, 2ND ED. by Robert McKim, 1980, Brooks/Cole.

GREAT MOMENTS IN MATHEMATICS AFTER 1650 by Howard W. Eves, 1982, Mathematical Association of America.

GREAT MOMENTS IN MATHEMATICS BEFORE 1650 by Howard W. Eves, 1982, Mathematical Association of America.

GEOMETRY: AN INVESTIGATIVE APPROACH by Phares G. O'Daffer and Stanley R. Clemens, 1977, Addison Wesley.

HISTORICAL TOPICS IN ALGEBRA, part of the 1971 Yearbook, National Council of Teachers of Mathematics.

IN MATHEMATICAL CIRCLES (2 Volumes) by Howard W. Eves, 1969, Prindle, Weber and Schmidt.

AN INTRODUCTION TO THE HISTORY OF MATHEMATICS by Howard W. Eves, 1983, Saunders College Publishing.

MATHEMATICAL CHALLENGES II - PLUS SIX, edited by Thomas Hill, 1974, National Council of Teachers of Mathematics.

MATHEMATICAL CIRCLES ADIEU by Howard W. Eves, 1977, Prindle, Weber and Schmidt.

MATHEMATICAL CIRCLES REVISITED by Howard W. Eves, 1971, Prindie, Weber and Schmidt.

MATHEMATICAL CIRCLES SQUARED by Howard W. Eves, 1972, Prindle, Weber and Schmidt.

MATHEMATICAL IDEAS by Charles D. Miller and Vern E. Heeron, 1980, Scott, Foresman.

MATHEMATICS CONTEST PROBLEMS FOR JUNIOR AND SENIOR HIGH SCHOOL, Saint Mary's College/Brother Alfred Brousseau, 1971, Creative Publications.

MATHEMATICS - PROBLEM SOLVING THROUGH RECREATIONAL MATHEMATICS by Bonnie Averbach and Orin Chein, 1980, W. H. Freeman.

MATHEMATICS TEACHER, published nine times yearly by the National Council of Teachers of Mathematics.

MATHEMATICS, THE MAN-MADE UNIVERSE, by Sherman K. Stein, 1969, W. H. Freeman.

THE NATURE OF MATHEMATICS by Karl J. Smith, 1984, Brooks/Cole Publishing Co.

NUMBERS AND INFINITY by Sondheimer and Rogerson, 1981, Cambridge University Press.

POLYHEDRAL MODELS by Magnus Wenninger, 1971, Cambridge University Press.

SIXTY CHALLENGING PROBLEMS WITH BASIC SOLUTIONS by Donald Spen , 1979, Hayden Book Co.

A SOURCEBOOK OF APPLICATIONS OF SCHOOL MATHEMATICS, prepared by Donald Bushaw and others, 1980, National Council of Teachers of Mathematics.

SPHERICAL MODELS by Magnus Wenninger, 1979, Cambridge University Press.

STATISTICS BY EXAMPLE by Frederick Mosteller et al, 1976, Addison Wesley.

TEACHING MATHEMATICS, A SOURCEBOOK OF AIDS, ACTIVITIES, AND STRATEGIES by Max A. Sobel and Evan M. Maletsky, 1975, Prentice-Hall.

TEACHING MATHEMATICS IN THE SECONDARY SCHOOL by Alfred S. Posamentier and Jay Stepelman, 1981, Charles E. Merrill.

TEACHING STATISTICS AND PROBABILITY, 1981 Yearbook, National Council of Teachers of Mathematics.

D. Other materials of general interest to both curriculum developers and classroom teachers are listed below:

AN AGENDA FOR ACTION - RECOMMENDATIONS FOR SCHOOL MATHEMATICS OF THE 1980s, 1980, National Council of Teachers of Mathematics.

THE AGENDA IN ACTION, 1983 Yearbook of the National Council of Teachers of Mathematics.

APPLICATIONS IN SCHOOL MATHEMATICS, 1979 Yearbook of the National Council of Teachers of Mathematics.

CLASSROOM IDEAS FROM RESEARCH ON SECONDARY SCHOOL MATHEMATICS by Donald Dessard and Marilyn Suydam, 1983, National Council of Teachers of Mathematics.

COMPUTERS IN MATHEMATICS, edited by David H. Ahl, 1979, Creative Computing Press.

COMPUTERS IN MATHEMATICS EDUCATION, 1984 Yearbook of the National Council of Teachers of Mathematics.

COMPUTERS IN TEACHING MATHEMATICS by P. Kelman et al, 1983, Addison Wesley.

COMPUTING AND MATHEMATICS: THE IMPACT ON SECONDARY SCHOOL MATHEMATICS, edited by James Fey, 1984, National Council of Teachers of Mathematics.

CONCEPTUAL BLOCKBUSTING by James L. Adams, 1980, W. W. Norton.

GUIDELINES FOR K-8 MATHEMATICS CURRICULUM, published by the Office of the Superintendent of Public Instruction, Olympia, WA, 1984.

THE HIGH SCHOOL MATHEMATICS LIBRARY by William L. Schaaf, 1982, National Council of Teachers of Mathematics.

HM MATH STUDY SKILLS PROGRAM by Catherine D. Tobin, 1980, National Council of Teachers of Mathematics.

HOW TO SOLVE IT by George Polya, 1957, Princeton University Press.

INDUCTION AND ANALOGY IN MATHEMATICS by George Polya, 1954, Princeton University Press.

MATH EQUALS by Terri Perl, 1978, Addison Wesley.

MATHEMATICAL DISCOVERY by George Polya, 1962, John Wiley and Sons.

MATHEMATICS: AN INTRODUCTION TO ITS SPIRIT AND USE, Readings from the Scientific American, 1979, W. H. Freeman.

MATHEMATICS PROJECTS HANDBOOK by Adrien L. Hess, 1982, National Council of Teachers of Mathematics.

PATTERNS OF PLAUSIBLE INFERENCE by George Polya, 1969, University of Princeton Press.

PROBLEM SOLVING IN SCHOOL MATHEMATICS, 1980 Yearbook of the National Council of Teachers of Mathematics.

SCHOOL MATHEMATICS: OPTIONS FOR THE 1990s, Volume I - Chairman's Report of a Conference cosponsored by the U.S. Department of Education, the National Council of Teachers of Mathematics and the Wisconsin Center for Education Research.

THE 10 BASIC SKILLS IN MATHEMATICS, Position Paper of the National Council of Supervisors of Mathematics, 1977.

10 ESSENTIAL IDEAS FOR BETTER TEACHING by Marilyn Suydam, Spring 1984 Special Edition of Washington Mathematics, Washington State Mathematics Council.

THINKING MATHEMATICALLY by John Mason et al, 1982, Addison Wesley.

USE EQUALS TO PROMOTE THE PARTICIPATION OF WOMEN IN MATHEMATICS by Alice Kaseberg, Nancy Kreinberg and Diane Downie, 1980, Lawrence Hall of Science, University of California.

WHAT SHOULD WE TEACH AS GEOMETRY? by Branko Grunbaum, Winter 1984-1985 Special Edition of Washington Mathematics, Washington State Mathematics Council.

III. TWO-YEAR SEQUENCE

A. CRITICAL THINKING AND PROBLEM SOLVING

STUDENTS SHOULD

	Year Introduced	Year(s) Developed	Year Mastered
1. Understand and use problem solving methods. Steps may include:			
a. formulating the problem;	1	12	
b. analyzing and conceptualizing the problem including the use of sketches, physical models, Venn diagrams, and estimation;	1	12	
c. identifying appropriate data and tools;	1	12	
d. using experimentation and intuition;	1	12	
e. transferring skills and strategies to new situations;	1	12	
f. drawing on background knowledge;	1	12	
g. modeling and predicting outcomes;	1	12	
h. finding and interpreting solutions;	1	12	
i. checking for reasonableness of results.	1	12	

Instructional Implications: From the earliest grade levels forward, teachers should expose students to situations for which the solution is not immediately obvious. Such problems should require exploration and result in knowledge acquisition. They may have multiple solutions, approximate solutions or both. They may, in fact, have no solution. The teacher should include problems which can be interpreted and approached in various ways. In this context, the teacher may wish to include both packaged computer programs and student-written programs to complement traditional problem-solving methods.

III-A. continued

2. Apply reasoning processes in problem solving that include:

a.	inductive reasoning;	1	12
b.	deductive reasoning.	1	12

Instructional Implications: The teacher must become increasingly sensitive to the frustration levels of students, allowing for the development of thinking skills to take place over extended periods of time. Further, teachers should be aware that the open-ended nature of problem solving and the associated thinking skills are such that closure is never reached. Teachers should take care to select activities that provide the excitement of discovery which in turn will lead to higher levels of motivation.

3. Understand and use probability and statistical methods in the problem-solving process, including attention to:

a.	collection of data;	1	12
b.	mean, median, mode;	1	12
c.	presentation and interpretation of data including the use of graphs, tables, etc.	1	12
d.	intuitive probability;	2	2
e.	odds.	2	2

Instructional Implications: Teachers should provide an abundance of "real-life" situations that involve the collection and interpretation of data. Teachers should also capitalize on the potential for the use of calculators and/or computers in facilitating the processing and displaying of data.

B. MEASUREMENT: SKILLS AND UNDERSTANDINGS

STUDENTS SHOULD

1. Apply the factor labeling concept (dimensional analysis): e.g., 50 km/hr x 2 hr is 100 km, etc.

1	12	

Instructional Implications: Teachers should introduce this concept as the need arises in finding solutions to problems. Activities should be provided that encourage an inductive approach in the discovery of the relationships that exist among the various units of measurement involved.

2. Use measuring devices in measurement activities involving length, area, volume, weight, angles, and time.

1	12	2

Instructional Implications: Teachers should provide numerous opportunities for hands-on measurement activities using such tools as rulers, compasses, protractors, balance and spring scales, and timing devices.

3. Understand rounding and the approximate nature of measurement and the limitations it imposes on computation.

1	12	2

Instructional Implications: Teachers should provide situations in which students compare results obtained by direct measurement activities such that they are led to an understanding of the approximate nature of measurement. Using the idea of "greatest possible error," students can compare maximum and minimum results obtained by computing areas and volumes.

4. Know the relationship among units within each system, English and metric; be able to convert measurements in given units to measurements in other units within each system; and be able to use "rule-of-thumb" methods to obtain approximate conversions between the systems (e.g., a meter is a little longer than a yard, etc.).

1	12	

Instructional Implications: Teachers should provide hands-on measurement activities that result in the discovery of the conversion factors. In particular, it is not sufficient to teach the "moving the decimal point" rules for converting among metric measurements; instead, experiential activities should provide students with the understandings that enable them to determine whether they are to multiply or divide by powers of ten (and by which powers of ten).

III-B. continued

5.	Be able to estimate length, weight, angles, and time.	1	12

Instructional Implications: Teachers should provide an array of measurement activities for which the only required outcomes are estimates. Students should also be encouraged to "guess and check" when engaging in measurement activities.

6.	Find indirect measurements using scale drawings and maps.	2	2

Instructional Implications: Teachers should provide opportunities for students to construct scale drawings and maps as well as to interpret existing drawings.

C. GEOMETRY: SKILLS AND UNDERSTANDINGS

STUDENTS SHOULD

1.	Understand and use the vocabulary and symbolism associated with simple geometric figures.	1	2

Instructional Implications: Teachers should provide instruction that will assist students in learning the terminology of geometry so that they will be able to describe properties of geometric figures in terms of basic definitions. Activities should be provided that involve the use of measurement and construction tools, diagrams, graphs, models, etc. to reinforce the learning of vocabulary in meaningful contexts.

2. Understand and use concepts related to plane and solid geometric figures such as:

a.	concepts of congruence using transformations;	2	2
b.	similarity;	2	2
c.	parallelism;	2	2
d.	perpendicularity;	2	2

e. skewness;	2	2	
f. symmetry.	2	2	

Instructional Implications: Teachers should provide opportunities for students to construct physical models that illustrate the various properties. Teachers should encourage students to look for examples within their environment.

3. Be able to use the Pythagorean Theorem.	2	2	

Instructional Implications: Teachers should provide "real life" applications involving triangles for which the measurement of the sides does not result in sets of Pythagorean triples. The use of calculators should be encouraged.

4. Demonstrate ability to perform basic geometric constructions such as constructing figures that are congruent to given figures, bisecting segments and angles, constructing perpendiculars and lines parallel to given lines, and constructions with paper folding.	2	2	

Instructional Implications: Teachers should provide for an integration of construction activities throughout the study of mathematics, thus facilitating transitions from the concrete to the abstract.

5. Be able to use coordinate geometry in practical applications.	2	2	

Instructional Implications: The teacher should provide applications of coordinate geometry such as simple linear graphs, map reading, grid games, etc.

D. NUMBER PROPERTIES: THEORY AND COMPUTATION

STUDENTS SHOULD

1. Be able to convert numbers between the different forms such as fractions to decimals, percents to fractions, etc.

1	12	2

Instructional Implications: Teachers should encourage the development of understanding through applications to "real life" situations.

2. Be able to arrange numbers in a sequential order.

1	12	2

Instructional Implications: Teachers should provide activities that require students to order rational numbers, negative as well as non-negative.

3. Be able to compute with rational numbers, and use rational approximations of square roots.

1	12	2

Instructional Implications: Teachers should provide an abundance of applications that require students to choose and use the correct computational processes. The use of calculators should be encouraged. Teachers should also provide instruction in the processes associated with signed numbers.

4. Be able to simplify numerical/algebraic expressions using the order of operations rule.

1	12	2

Instructional Implications: Teachers should provide opportunities for students to establish the rules through experiential activities. Students should be given opportunities and time to think through, discuss, and "defend" their conclusions.

5. Be able to simplify expressions using the properties of addition and multiplication such as commutative, associative, distributive, identity, and inverse properties.

1	12	2

Instructional Implications: Teachers should provide students with numerous examples that will help them to intuitively develop these concepts. Little emphasis should be placed on terminology.

III-D. continued

6. Be able to apply the transitive, the multiplicative, and the additive properties of equality and inequality.	1	12	2

Instructional Implications: Teachers should provide students with numerous examples that will help them to intuitively develop these concepts. Little emphasis should be placed on terminology.

7. Understand and use exponential/scientific notation.	1	12	

Instructional Implications: Teachers should provide activities designed to develop these concepts within the context of calculator problems involving large and small numbers. Emphasis needs to be placed on place value in the decimal system, but may be extended to include other bases.

E. ALGEBRA: CONCEPTS AND SKILLS

STUDENTS SHOULD

1. Demonstrate ability to work with simple algebraic expressions and formulas.	1	12	2

Instructional Implications: Teachers should provide students with numerous opportunities to benefit from experiences involving substituting values into practical formulas prior to consideration of abstract algebraic expressions. Neither should be attempted before students are reasonably adept at evaluating numerical expressions.

2. Solve and graph linear equations.	1	12	2

Instructional Implications: As much as possible, teachers should select activities involving solving and graphing linear equations from the "real world." Applications such as temperature scale conversions, banking formulas, etc. should be used.

III-E. continued

3. Understand and use ratio and proportion in solving problems including those involving:

1	12	

a. similarity;

b. intuitive probability.

Instructional Implications: Teachers should provide applications of ratio and proportion through the use of measurement and other hands-on activities.

F. FUNCTIONS

STUDENTS SHOULD

1. Understand the concept of function including attention to:

2	2	
1	12	

a. graphs of functions;

b. domain and range of functions.

Instructional Implications: Teachers should develop the concept of function through the use of concrete and representational illustrations and demonstrations such as a "function machine." Opportunities to look at many different kinds of functions should be provided. The use of sequences and patterns should be emphasized. Students should understand that a function can be represented by a formula, table, or graph. Example: A marathoner runs at an average rate of 325 meters per minute.

Formula

$d = 325t$

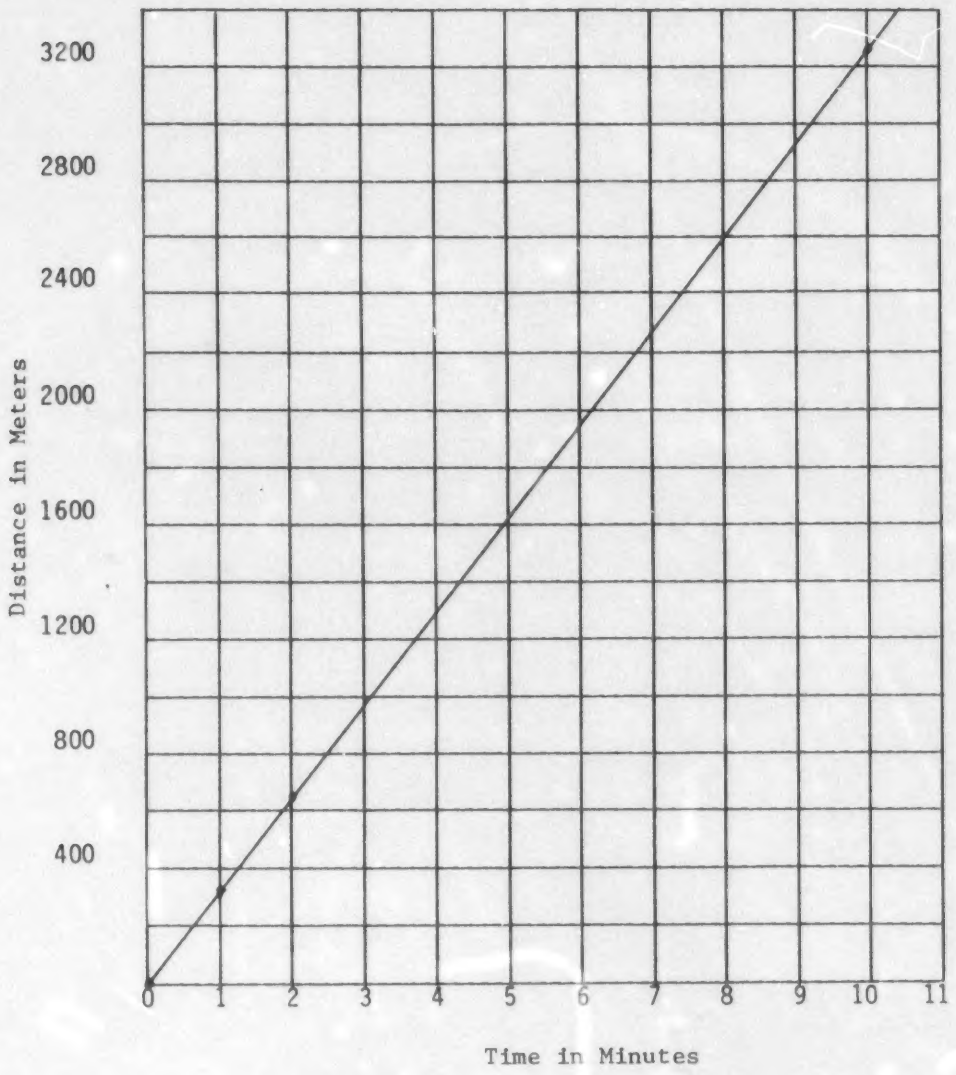
distance in meters

time in minutes

Table

t	0	1	2	3	...	10	...
d	0	325	650	975	...	3250	...

Graph


3200
2800
2400
2000
1600
1200
800
400
Distance in Meters
0
1
2
3
4
5
6
7
8
9
10
11
Time in Minutes

LIST OF SELECTED RESOURCES

The following listing of instructional resources is designed to provide curriculum developers and classroom teachers with a rich source of ideas and activities that support the goals and philosophy of the GUIDELINES FOR GRADES 9-12 MATHEMATICS CURRICULUM. The listing is not exhaustive; nor will everyone agree that each of the listed resources is of sufficient significance to merit its inclusion. Nevertheless, these are materials that have a number of knowledgeable mathematics educators have identified as eminently useful in their experience and which they believe will be valuable to those involved in the Guidelines implementation tasks that lie ahead.

A. The following publications contain activity pages that may be duplicated for student use and/or activity ideas that may easily be prepared for classroom implementation:

ACTIVITIES FROM THE MATHEMATICS TEACHER, edited by Evan M. Maletsky and Christian R. Hirsch, 1981, National Council of Teachers of Mathematics.

ALGEBRA IN THE CONCRETE by Mary Laycock and Reuben Schadler, 1973, Activity Resources Co.

ART "N" MATH by Karen Billings, Carol Campbell, and Alice Schwandt, 1975, Action Math Associates.

BASIC THINKING SKILLS SERIES (11 Booklets) by Anita Harnadek, 1977, Midwest Publications.

BRAIN STRETCHERS, BOOKS 1 and 2 by Carol Anderson and Jackie Haller, 1977, Midwest Publications

CLASSROOM QUICKIES, BOOKS 1, 2, and 3 by Anita Harnadek, 1978, Midwest Publications.

DEVELOPING SKILLS IN ESTIMATION, BOOK B by Dale Seymour, 1981, Dale Seymour Publications.

DEVELOPING THOUGHTFUL BEHAVIOR IN MATHEMATICS by John Firkins, 1983, Washington State Mathematics Council.

FIGURAL ANALOGIES (series of 3 booklets) by Howard Black and Sandra Black, 1982, Midwest Publications.

FIGURAL CLASSIFICATIONS (series of 3 booklets) by Howard Black and Sandra Black, 1983, Midwest Publications.

FIGURAL SEQUENCES (series of 3 booklets) by Howard Black and Sandra Black, 1982, Midwest Publications.

FIGURAL SIMILARITIES (series of 4 booklets) by Howard Black and Sandra Black, 1983, Midwest Publications.

FINITE DIFFERENCES, A PATTERN-DISCOVERY APPROACH TO PROBLEM SOLVING by Dale Seymour and Margaret Shedd, 1973, Creative Publications.

FOCUS ON GEOMETRY by Margaret Smart and Mary Laycock, 1982, Activity Resources Co.

HANDY MATH SERIES by Jan Fair

- FOCUS ON PURCHASING
- FOCUS ON TRAVEL
- FOCUS ON SPORTS
- FOCUS ON MAP SKILLS
- FOCUS ON EARNING MONEY
- FOCUS ON MANAGING MONEY

1979, Creative Publications.

INDUCTIVE THINKING SKILLS (series of 11 booklets) by Anita Harnadek, 1979, Midwest Publications.

THE MATHEMATICS OF ISLAMIC ART, produced by the Metropolitan Museum of Art, 1979, National Council of Teachers of Mathematics.

MATHEMATICS RESOURCE PROJECT

- NUMBER SENSE AND ARITHMETIC SKILLS
- MATHEMATICS IN SCIENCE AND SOCIETY
- RATIO, PROPORTION AND SCALING
- GEOMETRY AND VISUALIZATION
- STATISTICS AND INFORMATION ORGANIZATION

1978, Creative Publications.

MIND BENDERS (series of 11 booklets) by Anita Harnadek, 1978, Midwest Publications.

THE MINNEAPOLIS GENERAL MATH PROJECT

- IN A WORD
- THE WHOLE OF IT
- A REASONABLY CLOSE ENCOUNTER
- THE SIZE OF IT
- CHECKING OUT PATTERNS
- PART AND PARCEL
- WATCH YOUR WALLET
- CHARTING YOUR COURSE

1983, Dale Seymour Publications.

MULTICULTURAL MATHEMATICS POSTERS AND ACTIVITIES developed by the Mathematics Office, Seattle Public Schools, 1980, National Council of Teachers of Mathematics.

A NEW TWIST - DEVELOPING ARITHMETIC SKILLS THROUGH PROBLEM SOLVING by Jean J. Pedersen and Franz O. Armbruster, 1979, Addison Wesley.

NOTABLE NUMBERS by William Stokes, 1972, Creative Publications.

PROBLEM SOLVING, A HANDBOOK FOR TEACHERS by Stephen Krulik and Jesse A. Rudnick, 1980, Allyn and Bacon.

PROBLEM SOLVING IN MATHEMATICS, GRADE 8, Lane County Mathematics Project, 1983, Dale Seymour Publications.

PROBLEM SOLVING IN MATHEMATICS, GRADE 9 ALGEBRA, Lane County Mathematics Project, 1983, Dale Seymour Publications.

PROJECT-A-PUZZLE by Richard D. Porter, 1978, National Council of Teachers of Mathematics.

QUIZZLES by Wayne Williams, 1980, Creative Publications.

A SOURCEBOOK FOR TEACHING PROBLEM SOLVING by Stephen Krulik and Jesse A. Rudnick, 1984, Allyn and Bacon.

SPIRIT MASTERS ON CALCULATORS IN GEOMETRY AND MEASUREMENT by Charles H. Lund, 1981, J. Westen Walch.

TEACHING PROBLEM-SOLVING STRATEGIES by Daniel T. Dolan and James Williamson, 1983 Addison Wesley.

THINK ABOUT IT, MATHEMATICS PROBLEMS OF THE DAY by Marcy Cook, 1982, Creative Publications.

THINKING THROUGH PROBLEMS by Barnabas Hughes, 1975, Creative Publications.

USING CALCULATORS TO DEVELOP THOUGHTFUL BEHAVIOR IN MATHEMATICS by Edward C. Beardslee, 1984, Washington State Mathematics Council.

WHEN ARE WE EVER GONNA HAVE TO USE THIS? by Hal Saunders, 1981, Dale Seymour Publications.

WORD PROBLEMS (series of 9 booklets) by Anita Harnadek, 1980, Midwest Publications.

B. The following publications are appropriate for direct use by students and/or may serve as idea "banks" for curriculum developers as well as classroom teachers:

AHA! GOTCHA, PARADOXES TO PUZZLE AND DELIGHT by Martin Gardner, 1982, W. H. Freeman.

AHA! INSIGHT by Martin Gardner, 1978, Scientific American, W.H. Freeman.

ALGEBRA THROUGH APPLICATIONS, PART 1 and PART 2 by Zalman Usiskin, 1979, National Council of Teachers of Mathematics.

CREATING ESCHER-TYPE DRAWINGS by E. R. Ranucci and J. L. Teeters, 1977, Creative Publications.

CREATIVE CONSTRUCTIONS by Dale Seymour and Reuben Schadler, 1974, Creative Publications.

THE DOME BUILDERS HANDBOOK by John Prenis, 1973, Philadelphia Running Press.

THE DOT AND THE LINE by Norton Juster, 1963, Random House.

EUREKA by Dale Seymour and Richard Gidley, 1968, Creative Publications.

THE GRAPHIC WORK OF M. C. ESCHER by M. C. Escher, 1977, Ballantine Books.

HOW TO LIE WITH STATISTICS by Darrell Huff and Irving Geis, 1954, W. W. Norton.

HOW TO TAKE A CHANCE by Darrell Huff and Irving Geis, 1964, W. W. Norton.

I'M A NUMBER GAME by John Gregory and Dale Seymour, 1978, Creative Publications.

LIMERICK NUMBER PUZZLES by John Gregory and Dale Seymour, 1978, Creative Publications.

MATHEMATICAL FUN, GAMES AND PUZZLES by Jack Firohlichstein, 1967, Dover Publications.

MATHEMATICS, A HUMAN ENDEAVOR by Harold R. Jacobs, 1982, W. H. Freeman.

NUMBER TREASURY by Stanley Bezuszka and Margaret Kenny, 1982, Dale Seymour Publications.

150 PUZZLES IN CRYPT-ARITHMETIC by Maxey Brooke, 1969, Dover Publications.

PATTERNS IN SPACE by Robert Beard, 1973, Creative Publications.

POLYHEDRA PRIMER by Peter Pearce and Susan Pearce, 1978, Dale Seymour Publications.

POLYHEDRON MODELS FOR THE CLASSROOM by Magnus J. Wenninger, 1966, National Council of Teachers of Mathematics.

WOMEN IN MATHEMATICS by Lynn M. Osen, 1974, The MIT Press.

C. The following publications provide teachers with rich sources of problems and activity ideas for use with their students:

APPLICATIONS OF SECONDARY SCHOOL MATHEMATICS by Bernice Kastner, 1978, National Council of Teachers of Mathematics.

CREATIVE PROBLEM SOLVING IN MATHEMATICS by George Lechner, 1983, Houghton Mifflin.

GEOMETRY: AN INVESTIGATIVE APPROACH by Phares G. O'Daffer and Stanley R. Clemens, 1977, Addison Wesley.

IN MATHEMATICAL CIRCLES (2 Volumes) by Howard W. Eves, 1969, Prindle, Weber and Schmidt.

MATHEMATICAL CIRCLES ADIEU by Howard W. Eves, 1977, Prindle, Weber and Schmidt.

MATHEMATICAL CIRCLES REVISITED by Howard W. Eves, 1971, Prindle, Weber and Schmidt.

MATHEMATICAL CIRCLES SQUARED by Howard W. Eves, 1972, Prindle, Weber and Schmidt.

MATHEMATICS CONTEST PROBLEMS FOR JUNIOR AND SENIOR HIGH SCHOOL, Saint Mary's College/Brother Alfred Brousseau, 1971, Creative Publications.

MATHEMATICS TEACHER, published nine times yearly by the National Council of Teachers of Mathematics.

MEASUREMENT IN SCHOOL MATHEMATICS, 1976 Yearbook, National Council of Teachers of Mathematics.

NUMBERS AND INFINITY by Sondheimer and Rogerson, 1981, Cambridge University Press.

A SOURCEBOOK OF APPLICATIONS OF SCHOOL MATHEMATICS, prepared by Donald Bushaw and others, 1980, National Council of Teachers of Mathematics.

SUCCESSFUL PROBLEM SOLVING TECHNIQUES by Carole Greenes, John Gregory, and Dale Seymour, 1977, Creative Publications.

TEACHING MATHEMATICS, A SOURCEBOOK OF AIDS, ACTIVITIES, AND STRATEGIES by Max A. Sobel and Evan M. Maletsky, 1975, Prentice-Hall.

TEACHING MATHEMATICS IN THE SECONDARY SCHOOL by Alfred S. Posamentier and Jay Stepelman, 1981, Charles E. Merrill.

TEACHING STATISTICS AND PROBABILITY, 1981 Yearbook, National Council of Teachers of Mathematics.

D. Other materials of general interest to both curriculum developers and classroom teachers are listed below:

AN AGENDA FOR ACTION - RECOMMENDATIONS FOR SCHOOL MATHEMATICS OF THE 1980s, 1980, National Council of Teachers of Mathematics.

THE AGENDA IN ACTION, 1983 Yearbook of the National Council of Teachers of Mathematics.

APPLICATIONS IN SCHOOL MATHEMATICS, 1979 Yearbook of the National Council of Teachers of Mathematics.

CLASSROOM IDEAS FROM RESEARCH ON SECONDARY SCHOOL MATHEMATICS by Donald Dessard and Marilyn Suydam, 1983, National Council of Teachers of Mathematics.

COMPUTERS IN MATHEMATICS, edited by David H. Ahl, 1979, Creative Computing Press.

COMPUTERS IN MATHEMATICS EDUCATION, 1984 Yearbook of the National Council of Teachers of Mathematics.

COMPUTERS IN TEACHING MATHEMATICS by P. Kelman et al, 1983, Addison Wesley.

COMPUTING AND MATHEMATICS: THE IMPACT ON SECONDARY SCHOOL MATHEMATICS, edited by James Fey, 1984, National Council of Teachers of Mathematics.

CONCEPTUAL BLOCKBUSTING by James L. Adams, 1980, W. W. Norton.

GUIDELINES FOR K-8 MATHEMATICS CURRICULUM, published by the Office of the Superintendent of Public Instruction, Olympia, WA, 1984.

THE HIGH SCHOOL MATHEMATICS LIBRARY by William L. Schaaf, 1982, National Council of Teachers of Mathematics.

HM MATH STUDY SKILLS PROGRAM by Catherine D. Tobin, 1980, National Council of Teachers of Mathematics.

HOW TO SOLVE IT by George Polya, 1957, Princeton University Press.

INDUCTION AND ANALOGY IN MATHEMATICS by George Polya, 1954, Princeton University Press.

MATH EQUALS by Terri Perl, 1978, Addison Wesley.

MATHEMATICAL DISCOVERY by George Polya, 1962, John Wiley and Sons.

MATHEMATICS: AN INTRODUCTION TO ITS SPIRIT AND USE, Readings from the Scientific American, 1979, W. H. Freeman.

MATHEMATICS PROJECTS HANDBOOK by Adrien L. Hess, 1982, National Council of Teachers of Mathematics.

PATTERNS OF PLAUSIBLE INFERENCE by George Polya, 1969, University of Princeton Press.

PROBLEM SOLVING IN SCHOOL MATHEMATICS, 1980 Yearbook of the National Council of Teachers of Mathematics.

SCHOOL MATHEMATICS: OPTIONS FOR THE 1990s, Volume I - Chairman's Report of a Conference cosponsored by the U.S. Department of Education, the National Council of Teachers of Mathematics and the Wisconsin Center for Education Research.

THE 10 BASIC SKILLS IN MATHEMATICS, Position Paper of the National Council of Supervisors of Mathematics, 1977.

10 ESSENTIAL IDEAS FOR BETTER TEACHING by Marilyn Suydam, Spring 1984 Special Edition of Washington Mathematics, Washington State Mathematics Council.

THINKING MATHEMATICALLY by John Mason et al, 1982, Addison Wesley.

USE EQUALS TO PROMOTE THE PARTICIPATION OF WOMEN IN MATHEMATICS by Alice Kaseberg, Nancy Kreinberg and Diane Downie, 1980, Lawrence Hall of Science, University of California.

WHAT SHOULD WE TEACH AS GEOMETRY? by Branko Grunbaum, Winter 1984-1985 Special Edition of Washington Mathematics, Washington State Mathematics Council.

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IPS/624/85